kd education academy (9582701166)

Time : 6 Hour | STD 11 Maths | Total Marks : 304

kd 90+ ch- 10 conic sections basic to jee

*** Match the following.** **[10]**

1.

Part (A)	Part (B)
1. The length of latus rectum of parabola $x^2 = 4ay$	(a) $2a$
2. The length of major axis of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	(b) $(\pm a, 0)$
3. The coordinates of vertex of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	(c) $y = b/e, y = -b/e$
4. The equation of directrix of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$	(d) $(-a, 0)$
5. The coordinates of focus of parabola $y^2 = -4ax$	(e) $4a$

Ans. : 1.(e), 2.(a), 3.(b), 4.(c), 5.(d)

2.

Part (A)	Part (B)
1. The coordinates of focus of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$	(a) $(-g, -f)$
2. The equation of major axis of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	(b) $(0, \pm be)$
3. The length of latus rectum of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$	(c) $y = 0$
4. The coordinates of centre of circle $x^2 + y^2 + 2gx + 2fy + c = 0$	(d) $2b$
5. The length of major axis of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$	(e) $\frac{2a^2}{b}$

Ans. : 1.(b), 2.(c), 3.(a), 4.(e), 5.(d)

*** Choose the right answer from the given options. [1 Marks Each]** **[148]**

3. If the equation $\frac{\lambda(\mathrm{x}+1)^2}{3} + \frac{(\mathrm{y}+2)^2}{4} = 1$ represents a circle then λ:

(A) 1 (B) $\frac{3}{4}$ (C) 0 (D) $-\frac{3}{4}$

Ans. :

b. $\frac{3}{4}$

Solution:

$\frac{\lambda(\mathrm{x}+1)^2}{3} + \frac{(\mathrm{y}+2)^2}{4} = 1$

for a circle of a $(\mathrm{x} - \alpha)^2 + 6(\mathrm{y} - \beta)^2 = 1$ then (a = 6)

$\frac{\lambda}{3} = \frac{1}{4}$

$\lambda = \frac{3}{4}$

4. The length of the latus-rectum of the parabola x^2 - 4x - 8y + 12 = 0 is

(A) 4 (B) 6 (C) 8 (D) 10

Ans. :

c. 8

Solution:

Given:

$x^2 - 4x - 8y + 12 = 0$

$(x - 2)^2 - 8y + 8 = 0$

$(x - 2)^2 = 8y - 8 = 8(y - 1)$

Let X = x - 2, Y = y - 1

$\therefore X^2 = 8Y$

$\therefore$ Length of the latus rectum = 4a = 8 units

5. If the circles $x^2 + y^2 = a$ and $x^2 + y^2 - 6x - 8y + 9 = 0$, touch externally, then a =

(A) 1 (B) -1 (C) 21 (D) 16

Ans. :

a. 1

Solution:

$x^2 + y^2 = a$ .... (1)

And, $x^2 + y^2 - 6x - 8y + 9 = 0$ .... (2)

Let circles (1) and (2) touch each other at point P.

The centre of the circle $x^2 + y^2 = a$, 0, is (0, 0).

The centre of the circle $x^2 + y^2 - 6x - 8y + 9 = 0$, C_1, is (3, 4).

Also, radius of circle (1) $= \sqrt{a} = OP$

Radius of circle (2) $\sqrt{9 + 16 - 9} = 4 = C_1P$

From figure, we have:

$\Rightarrow \sqrt{3^2 + 4^2} = 4 + \sqrt{a}$

$\Rightarrow 5 = 4 + \sqrt{a}$

$\Rightarrow a = 1$

6. Choose the correct answer.

The distance between the foci of a hyperbola is 16 and its eccentricity is 2. Its equation is:

(A) $x^2 - y^2 = 32$ (B) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (C) $2x - 3y^2 = 7$ (D) none of these.

Ans. :

a. $x^2 - y^2 = 32$

Solution:

We know that distance between the foci = 2ae

$\therefore\ 2ae = 16$

$\Rightarrow ae = 8$

Given that $e = \sqrt{2}$

$\therefore\ \sqrt{2}a = 8$

$\Rightarrow a = 4\sqrt{2}$

Now, $b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = 32(32 - 1)$

$\Rightarrow b^2 = 32$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1$

$\Rightarrow x^2 - y^2 = 32$

7. If the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other, then:

(A) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$ (B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$ (C) $a + b = 2c$ (D) $\frac{1}{a} + \frac{1}{b} = \frac{2}{c}$

Ans. :

a. $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$

Solution:

Given:

$x^2 + y^2 + 2ax + c = 0$ .... (1)

And, $x^2 + y^2 + 2by + c = 0$ .... (2)

For circle (1), we have:

Centre = (-a, 0) = C_1

For circle (2), we have:

Centre = (0,-b) = C_2

Let the circles intersect at point P.

$\therefore$ Coordinates of P = Mid point of C_1C_2

$\Rightarrow$ Coordinates of P $= \left(\frac{-a+0}{2},\ \frac{0-b}{2}\right) = \left(\frac{-a}{2},\ \frac{-b}{2}\right)$

Now, we have:

PC_1 = radius of (1)

$\Rightarrow \sqrt{(-a + \frac{a}{2})^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{a^2 - c}$

$\Rightarrow \frac{a^2}{4} + \frac{b}{4}^2 = a^2 - c \ldots\ldots (3)$

Also, radius of circle (1) = radius of circle (2)

$\Rightarrow \sqrt{a^2 - c} = \sqrt{b^2 - c}$

$\Rightarrow a^2 = b^2 \ldots\ldots (4)$

From (3) and (4), we have:

$\frac{a^2}{2} = a^2 - c$

$\Rightarrow \frac{a^2}{2} = c$

$\Rightarrow \frac{2}{a^2} = \frac{1}{c}$

$\Rightarrow \frac{1}{a^2} + \frac{1}{a^2} = \frac{1}{c}$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$

8. The center of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$ is:

(A) (2, -3) (B) (-2, 3) (C) (-4, 6) (D) (4, -6)

Ans. :

a. (2, -3)

9. If the point $(\lambda, \lambda+1)$ lies inside the region bounded by the curve $x = \sqrt{25 - y^2}$ and y-axis, then λ belongs to the interval:

(A) $(-1, 3)$ (B) $(-4, 3)$

(C) $(-\infty, -4) \cup (3, \infty)$ (D) None of these

Ans. :

a. $(-1, 3)$

Solution:

The given equation of the curve is $x^2 + y^2 = 25$

Since $(\lambda, \lambda+1)$ lies inside the region bounded by the curve $x^2 + y^2 = 25$ and the y-axis, we have:

$\lambda^2 + (\lambda+1)^2 < 25,$ provided $\lambda + 1 > 0$

$\Rightarrow \lambda^2 + \lambda^2 + 12\lambda < 25, \lambda > -1$

$\Rightarrow 2\lambda^2 + 2\lambda - 24 < 0, \lambda > -1$

$\Rightarrow \lambda^2 + \lambda - 12 < 0, \lambda > -1$

$\Rightarrow (\lambda - 3)(\lambda + 4) < 0, \lambda > -1$

$\Rightarrow -4 < \lambda < 3, \lambda > -1$

$\Rightarrow \lambda \in (-1, 3)$

10. The equation $x^2 + y^2 - 2x + 4y + 5 = 0$ represents:

(A) A point (B) A pair of straight lines

(C) A circle of non zero radius (D) None of these

Ans. :

a. A point

Solution:

$x^2 + y^2 - 2x + 4y + 5 = 0$

$(x - 1)^2 + (y + 2)^2 - 5 + 5 = 0$

$\Rightarrow (x - 1)^2 + (y + 2)^2 = 0$

Since, radius is 0, its a point

Alternative method:

Here, a = b = 1

$r = \sqrt{1+4-5=0}$

a circle of radius 0. So, its a point.

11. The radius of the circle represented by the equation $3x^2+3y^2+(\lambda-6)y+3=0$ is:

(A) $\frac{3}{2}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\frac{2}{3}$ (D) None of these

Ans. :

a. $\frac{3}{2}$

Solution:

The equation of the circle is $3x^2+3y^2+(\lambda-6)y+3=0$

$\therefore$ Coefficient of $xy=0$

$\Rightarrow \lambda=0$

$\therefore 3x^2+3y^2+9x-6y+3=0$

$\Rightarrow x^2+y^2+3x-2y+1=0$

Therefore, the radius of the circle is $\sqrt{\left(\frac{3}{2}\right)^2+(-1)^2-1} = \frac{3}{2}$.

12. If the circle x^2 + y^2 + 2ax + 8y + 16 = 0 touches x-axis, then the value of a is:

(A) ±16 (B) ±4 (C) ±8 (D) ±1

Ans. :

b. ±4

Solution:

The equation of the circle is x^2 + y^2 + 2ax + 8y + 16 = 0.

Its centre is (-a, -4) and its radius is a units.

Since the circle touches the x-axis, we have:

$\sqrt{(-a+a)^2+(4-0)^2} = a$

$\Rightarrow a=\pm4$

13. If $2x^2+\lambda xy+2y^2(\lambda-4)x+6y-5=0$ is the equation of a circle, then its radius is:

(A) $3\sqrt{2}$ (B) $2\sqrt{3}$ (C) $2\sqrt{2}$ (D) None of these

Ans. :

d. None of these

Solution:

The given equation is $2x^2+\lambda xy+2y^2+(\lambda-4)x+6y-5=0$ which can be rewritten as

$x^2+\frac{\lambda xy}{2}+y^2+\frac{(\lambda-4)}{2}x+3y-\frac{5}{2}=0.$

Comparing the given equation $x^2+y62+2gx+2fy+c=0$ with we get: $\lambda=0$

$\therefore x^2+y^2-2x+3y-\frac{5}{2}=0$

$\therefore$ Radius $=\sqrt{(-1)^2+\left(\frac{3}{2}\right)^2+\frac{5}{2}} = \sqrt{1+\frac{9}{4}+\frac{5}{2}} = \sqrt{\frac{23}{4}} = \frac{\sqrt{23}}{2}$

14. Equation of the diameter of the circle $x^2 + y^2 - 2x + 4y = 0$ which passes through the origin is:

(A) $x + 2y = 0$ (B) $x - 2y = 0$ (C) $2x + y = 0$ (D) $2x - y = 0$

Ans. :

c. $2x + y = 0$

Solution:

Let the diameter of the circle be y = mx.

Since the diameter of the circle passes through its centre, (1, -2) satisfies the equation of the diameter.

$\therefore$ m = -2

Substituting the value of m in the equation of diameter:

y = -2x

$\Rightarrow$ 2x + y = 0

Hence, the required equation of the diameter is 2x + y = 0.

15. Determine the area enclosed by the curve $x^2 - 10x + 4y + y^2 = 196$:

(A) 15π (B) 225π (C) 20π (D) 17π

Ans. :

b. 225π

16. Choose the correct answer.

The area of the circle centred at (1, 2) and passing through (4, 6) is:

(A) 5π (B) 10π (C) 25π (D) none of these.

Ans. :

c. 25π

Solution:

Given that the centre of the circle is (1, 2)

Radius of the circle $= \sqrt{(4-1)^2 + (6-2)^2}$

$= \sqrt{9+16} = 5$

So, the area of the circle $= \pi r^2$

$= \pi \times (5)^2 = 25\pi$

17. The circle with radius 1 and centre being foot of the perpendicular from (5, 4) on y-axis, is:

(A) $x^2 + y^2 - 8x - 15 = 0$ (B) $x^2 + y^2 - 10x + 24 = 0$

(C) $x^2 + y^2 - 8y + 15 = 0$ (D) $x^2 + y^2 + 2y = 0$

Ans. :

c. $x^2 + y^2 - 8y + 15 = 0$

Solution:

Foot of perpendicular of (5, 4) on y-axis is (0, 4)

$\therefore$ The equation of circle with

radius 1cm is $(x - 0)^2 + (y - 4)^2 = 1$

$\Rightarrow x^2 + y^2 - 8y + 16$

$\Rightarrow x^2 + y^2 - 8y + 15 = 0$

18. The equations of the tangents to the ellipse $9x^2 + 16y^2 = 144$ from the point (2, 3) are:

(A) y = 3, x = 5 (B) x = 2, y = 3 (C) x = 3, y = 2 (D) x + y = 5, y = 3

Ans. :

d. x + y = 5, y = 3

$9x^2 + 16y^2 = 144$

Solution:

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$

Equation of the tangent in case of an ellipse is given by

$y = mx + \sqrt{a^2m^2 + b^2}$

$\Rightarrow y = mx + \sqrt{16m^2 + 9}\ \ldots(1)$

Substituting x = 2 and y = 3, we get:

$3 = 2m \pm \sqrt{16m^2 + 9}$

$\Rightarrow 3 - 2m = \sqrt{16m^2 + 9}$

On squaring both sides, we get:

$(3 - 2m)^2 = (16m^2 + 9)$

$\Rightarrow 9 + 4m^2 - 12m = (16m^2 + 9)$

$\Rightarrow 12m^2 + 12m = 0$

$\Rightarrow 12m(m+1) = 0$

$\Rightarrow m = 0, -1$

Substituting values of m in eq. (1), we get:

For $m = 0,\ y = 3$

For $m = -1,\ y = -x + 5$ or $x + y = 5$

19. If the focus of a parabola is (-2, 1) and the directrix has the equation x + y = 3, then its vertex is

(A) $(0, 3)$ (B) $\left(-1,\ \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $(2, -1)$

Ans. :

c. $(-1, 2)$

Solution:

Given:

The focus S is at (-2, 1) and the directrix is the line x + y - 3 = 0.

The slope of the line perpendicular to x + y - 3 = 0 is 1.

The axis of the parabola is perpendicular to the directrix and passes through the focus.

$\therefore$ Equation of the axis of the parabola = y - 1 = 1(x + 2) ...(1)

Intersection point of the directrix and the axis is the intersection point of (1) and x + y - 3 = 0.

Let the intersection point be K.

Therefore, the coordinates of K will be (0, 3).

Let (h, k) be the coordinates of the vertex, which is the mid-point of the segment joining K and the focus.

$\therefore\ h=\frac{0-2}{2},\ k=\frac{3+1}{2}$

h = -1, k = 2

Hence, the coordinates of the vertex are $(-1, 2)$.

20. If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 8y + c = 0$ touch each other, then c is equal to:

(A) 15 (B) -15 (C) 16 (D) -16

Ans. :

a. 15

Solution:

The centre of the circle $x^2 + y^2 = 9$ is (0, 0).

Let us denote it by C_1.

The centre of the circle $x^2 + y^2 + 8y + c = 0$ is (0, -4).

Let us denote it by C_2.

The radius of $x^2 + y^2 = 9$ is 3 units.

$x^2 + y^2 + 8y + c = 0$

$\Rightarrow (x-0)^2+(y+4)^2=16-c=(\sqrt{16-c})^2$

Therefore, the radius of the above circle is $\sqrt{16-c}$

Let the circles touch each other at P.

$\therefore C_1C_2=PC_2+PC_1$

$\Rightarrow PC_2=4-3=1$

$\Rightarrow PC_2-1=\sqrt{16-c}$

$\Rightarrow c=15$

21. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is:

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) 4

Ans. :

b. $\frac{4}{3}$

Solution:

Since, the parabola $y^2 = 4ax$ passes through the point (3, 2)

$\Rightarrow 2^2 = 4a \times 3$

$\Rightarrow 4 = 12a$

$\Rightarrow \mathrm{a} = \frac{4}{12}$

$\Rightarrow \mathrm{a} = \frac{1}{3}$

So, the length of latusrectum $= 4\mathrm{a} = 4 \times (\frac{1}{3}) = \frac{4}{3}$

22. The vertex of the parabola $x^2 + 8x + 12y + 4 = 0$ is

(A) (-4, 1) (B) (4, -1) (C) (-4, -1) (D) (4, 1)

Ans. :

a. (-4, 1)

Solution:

Given:

$x^2 + 8x + 12y + 4 = 0$

$\Rightarrow (x+4)^2 - 16 + 12y + 4 = 0$

$\Rightarrow (x+4)^2 + 12y - 12 = 0$

$\Rightarrow (x+4)^2 = -12(y - 1)$

Let X = x + 4, Y = y - 1

$X^2 = -12Y$

Vertex = (X = 0,Y = 0) = (x + 4 = 0,y - 1 = 0) = (x = -4,y = 1)

Hence, the vertex is at (-4, 1).

23. The focus of the parabola $y = 2x^2 + x$ is

(A) $(0,0)$ (B) $\left(\frac{1}{2},\frac{1}{4}\right)$ (C) $\left(-\frac{1}{4},0\right)$ (D) $\left(-\frac{1}{4},\frac{1}{8}\right)$

Ans. :

c. $\left(-\frac{1}{4},0\right)$

Solution:

Given:

Equation of the parabola = $y = 2x^2 + x$

$\Rightarrow \mathrm{x}^2 + \frac{\mathrm{x}}{2} = \frac{\mathrm{y}}{2}$

$\Rightarrow \left(\mathrm{x} + \frac{1}{4}\right)^2 = \frac{\mathrm{y}}{2} + \frac{1}{16}$

$\Rightarrow \left(\mathrm{x} + \frac{1}{4}\right)^2 = \frac{8\mathrm{y}+1}{16}$

$\Rightarrow \left(\mathrm{x} + \frac{1}{4}\right)^2 = \frac{1}{2}(\mathrm{y} + \frac{1}{8})$

Let $\mathrm{X} = \mathrm{x} + \frac{1}{4}, \mathrm{Y} = \mathrm{y} + \frac{1}{8}$

$\therefore\ \mathrm{X}^2 = \frac{1}{2}\mathrm{Y}$

Comparing with X = 4aY

$\mathrm{a} = \frac{1}{8}$

Focus $= (\mathrm{X} = 0,\ \mathrm{Y} = \mathrm{a}) = \left(\mathrm{x} = \frac{-1}{4}, \mathrm{y} = 0\right)$

Hence, the focus is at $\left(-\frac{1}{4}, 0\right)$.

24. If the circle $x^2 + y^2 = 9$ passesthrough (2, c) then c is equal to:

(A) $\sqrt{5}$ (B) $\sqrt{6}$ (C) $\sqrt{3}$ (D) $\sqrt{7}$

Ans. :

a. $\sqrt{5}$

Solution:

The equation of circle $x^2 + y^2 = 9$ The point is (2, c)

$\Rightarrow 2^2 + c^2 = 9$

$4 + c^2 = 9$

$c^2 = 9 - 4$

$c^2 = 5$

$\mathrm{c} = \sqrt{5}$

25. The coordinates of the focus of the parabola $y^2 - x - 2y + 2 = 0$ are

(A) $\left(\frac{5}{4}, 1\right)$ (B) $\left(\frac{1}{4}, 0\right)$ (C) $(1, 1)$ (D) None of these

Ans. :

a. $\left(\frac{5}{4}, 1\right)$

Solution:

Given:

The equation of the parabola is $y^2 - x - 2y + 2 = 0$.

$\Rightarrow$ (y - 1) - 1 = (x - 2)

(y - 1) = x - 1

Let X = x - 1, Y = y - 1

Y = X

Comparing with Y = 4aX:

$\mathrm{a} = \frac{1}{4}$

Focus=

$(\mathrm{X} = \mathrm{a}, \mathrm{Y} = 0) = (\mathrm{X} = \frac{1}{4}, \mathrm{Y} = 0) = (\mathrm{x} = \frac{1}{4} + 1, \mathrm{y} = 1)$

$= (\mathrm{x} = \frac{5}{4}, \mathrm{y} = 1)$

Hence, the focus is at $\left(\frac{5}{4}, 1\right)$

26. If V and S are respectively the vertex and focus of the parabola $y^2 + 6y + 2x + 5 = 0$, then SV =

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) None of these

Ans. :

b. $\frac{1}{2}$

Solution:

Given:

The vertex and the focus of a parabola are V and S, respectively.

The given equation of parabola can be rewritten as follows:

$(y + 3)^2 - 9 + 5 + 2x = 0$

$\Rightarrow (y + 3)^2 + 2x = 4$

$\Rightarrow (y + 3)^2 = 4 - 2x$

$\Rightarrow (y + 3)^2 = -2(x - 2)$

Let $Y = y + 3$, $X = x - 2$

Then, the equation of parabola becomes $Y^2 = -2X$.

Vertex = $(X = 0, Y = 0) = (x - 2 = 0, y + 3 = 0) = (x = 2, y = -3)$

Comparing with $y^2 = 4ax$:

$4a = 2 \Rightarrow a = \frac{1}{2}$

Focus = $\left(X = \frac{-1}{2}, Y = 0\right) = \left(x - 2 = \frac{-1}{2}, y + 3 = 0\right) = \left(x = \frac{3}{2}, y = -3\right)$

$\Rightarrow \ SV = \sqrt{\left(2 - \frac{3}{2}\right)^2 + (-3 + 3)^2} = \frac{1}{2}$

27. If the point (2, k) lies outside the circles $x^2 + y^2 + x - 2y - 14 = 0$ and $x^2 + y^2 = 13$ then klies in the interval:

(A) $(-3, \ -2) \cup (3, \ 4)$ (B) $-3, \ 4$ (C) $(-\infty, \ -3) \cup (4, \ \infty)$ (D) $(-\infty, \ -2) \cup (3, \ \infty)$

Ans. :

c. $(-\infty, \ -3) \cup (4, \ \infty)$

Solution:

The given equations of the circles are $x^2 + y^2 + x - 2y - 14 = 0$ and $x^2 + y^2 = 13$.

Since (2, k) lies outside the given circles, we have:

$4 + k^2 + 2 - 2k - 14 > 0$ and $4 + k^2 > 13$

$\Rightarrow k^2 - 2k - 8 > 0$ and $k^2 > 9$

$\Rightarrow (k - 4)(k + 2) > 0$ and $k^2 > 9$

$\Rightarrow k > 4$ or $k < -2$ and $k > 3$ or $k < -3$

$\Rightarrow k > 4$ and $k < -3$

$\Rightarrow k \in (-\infty, \ -3) \cup (4, \ \infty)$

28. The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centres lie in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 =$

0, where c is equal to:

(A) 4 (B) 2 (C) 3 (D) 6

Ans. :

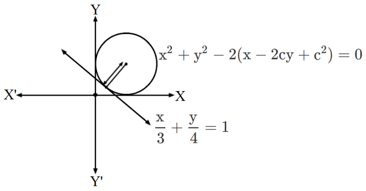
d. 6

Solution:

The equation of the circle that touches the axes of coordinates is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$.

Also, $x^2 + y^2 - 2cx - 2cy + c^2 = 0$ touches the line $\frac{x}{3} + \frac{y}{4} = 1$ or $4x + 3y - 12 = 0$.

Since the circle lies in the first quadrant, it centre is is (c, c).



From the figure, we have:

$$\left|\frac{4c+3c-12}{\sqrt{4^2+3^3}}\right| = c$$

$$\Rightarrow \frac{7c-12}{5} = c$$

$$\Rightarrow c = 6$$

29. The equation of the parabola whose vertex is (a, 0) and the directrix has the equation x + y = 3a, is

(A) $x^2 + y^2 + 2xy + 6ax + 10ay + 7a^2 = 0$

(B) $x^2 - 2xy + y^2 + 6ax + 10ay - 7a^2 = 0$

(C) $x^2 - 2xy + y^2 - 6ax + 10ay - 7a^2 = 0$

(D) None of these

Ans. :

b. $x^2 - 2xy + y^2 + 6ax + 10ay - 7a^2 = 0$

Solution:

Given:

The vertex is at (a, 0) and the directrix is the line x + y = 3a.

The slope of the line perpendicular to x + y = 3a is 1.

The axis of the parabola is perpendicular to the directrix and passes through the vertex.

∴ Equation of the axis of the parabola = y − 0 = 1(x - a) ...(1)

Intersection point of the directrix and the axis is the intersection point of (1) and x + y = 3a.

Let the intersection point be K.

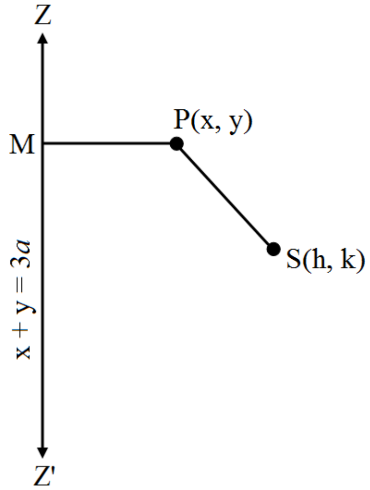
Therefore, the coordinates of K are (2a, a)

The vertex is the mid-point of the segment joining K and the focus (h, k).

$\therefore\ a = \frac{2a+h}{2},\ 0 = \frac{a+k}{2}$

h = 0, k = -a

Let P (x, y) be any point on the parabola whose focus is S (h, k) and the directrix is x + y= 3a.



Draw PM perpendicular to x + y = 3a.

Then, we have:

SP = PM

⇒ $SP^2 = PM^2$

$\Rightarrow\ (x-0)^2 + (y+a)^2 = \left(\frac{x+y-3a}{\sqrt{2}}\right)^2$

$\Rightarrow\ x^2 + (y+a)^2 = \left(\frac{x+y-3a}{\sqrt{2}}\right)^2$

$\Rightarrow\ 2x^2 + 2y^2 + 2a^2 + 4ay = x^2 + y^2 + 9a^2 + 2xy - 6ax - 6ay$

$\Rightarrow\ x^2 + y^2 - 7a^2 + 10ay + 6ax = 0$

30. The eccentricity of the conic $9x^2 + 25y^2 = 225$ is:

(A) $\frac{2}{5}$ (B) $\frac{4}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{5}$

Ans. :

b. $\frac{4}{5}$

$9x^2 + 25y^2 = 225$

Solution:

$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$

Comparing it with $\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ we get:

$a = 5$ and $b = 3$

Here, a > b, so the major and the minor axes of the ellipse are along the x–axis and y–axis, respectively.

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow \mathrm{e} = \sqrt{1 - \frac{9}{25}}$

$\Rightarrow \mathrm{e} = \sqrt{\frac{16}{25}}$

$\Rightarrow \mathrm{e} = \frac{4}{5}$

31. The equation of the circle passing through (3, 6) and whose centre is (2, -1) is:

(A) $x^2 + y^2 - 4x + 2y = 45$

(B) $x^2 + y^2 - 4x - 2y + 45 = 0$

(C) $x^2 + y^2 + 4x - 2y = 45$

(D) $x^2 + y^2 - 4x + 2y + 45 = 0$

Ans. :

a. $x^2 + y^2 - 4x + 2y = 45$

32. The circle $x^2 + y^2 - 3x - 4y + 2 = 0$ cuts x-axis:

(A) (2, 0), (-3, 0) (B) (3, 0), (4, 0) (C) (1, 0), (-1, 0) (D) (1, 0), (2, 0)

Ans. :

d. (1, 0), (2, 0)

Solution:

$x^2 + y^2 - 3x - 4y + 2 = 0$

x-axis will be cut when y = 0

put y=0

$x^2 - 3x + 2 = 0$

(x - 2) (x - 1) = 0

x = 1, 2

points (1, 0), (2, 0)

33. The equation of the incircle formed by the coordinate axes and the line 4x + 3y = 6 is:

(A) $x^2 + y^2 - 6x - 6y + 9 = 0$

(B) $4 (x^2 + y^2 - x - y) + 1 = 0$

(C) $4 (x^2 + y^2 + x + y) + 1 = 0$

(D) None of these

Ans. :

b. $4 (x^2 + y^2 - x - y) + 1 = 0$

Solution:

The line 4x + 3y = 6 cuts the coordinate axes at $\left(\frac{3}{2}, 0\right)$ and (0, 2)

The coordinates of the incentre is $\left(\frac{\mathrm{ax_1+bx_2+cx_3}}{\mathrm{a+b+c}}, \frac{\mathrm{ay_1+by_2+cy}}{\mathrm{a+b+c}}\right)$

Here, $\mathrm{a} = \frac{5}{2}, \mathrm{b} = \frac{3}{2}, \mathrm{c} = 2, \mathrm{x_1} = 0, \mathrm{y_1}$

$= 0, \mathrm{x_2} = 0, \mathrm{y_2} = 2, \mathrm{x_3} = \frac{3}{2}, \mathrm{y_3} = 0$

Thus, the coordinates of the incentre:

$\left(\frac{0+0+3}{6}, \frac{0+3+0}{6}\right)$

$= \left(\frac{1}{2}, \frac{1}{2}\right)$

The equation of the incircle:

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = a^2$

Also, radius of the incircle $= \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$

Here, $s = \frac{a+b+c}{2} = \frac{\frac{5}{2}+\frac{3}{2}+2}{2} = 3$

$\therefore$ Radius of the incircle $= \sqrt{\frac{3(3-a)(3-b(3-c))}{3}}$

$= \frac{\sqrt{3\left(3-\frac{5}{2}\right)\left(3-\frac{3}{2}\right)(3-c)}}{3}$

$= \frac{\sqrt{3\left(3-\frac{1}{2}\right)\left(\frac{3}{2}\right)}}{3}$

$= \frac{1}{2}$

The equation of circle:

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow 4(x^2 + y^2) - x - y + 1 = 0$

34. Find the equation of the circle.
Centered at (3, -2) with radius 4:

(A) $x^2 + y^2 + 6x - 4y = 3$
(B) $x^2 + y^2 - 6x + 4y = 3$
(C) $x^2 + y^2 - 3x + 2y = -3$
(D) $x^2 + y^2 + 3x - 2y = -3$

Ans. :

b. $x^2 + y^2 - 6x + 4y = 3$

35. The eccentricity of the ellipse, if the distance between the foci is equal to the length of the latus-rectum, is:

(A) $\frac{\sqrt{5}-1}{2}$
(B) $\frac{\sqrt{5}+1}{2}$
(C) $\frac{\sqrt{5}-1}{4}$
(D) none of these

Ans. :

a. $e = \frac{\sqrt{5}-1}{2}$

Solution:

According to the question, the distance between the foci is equal to the lengt h of the latus rectum.

$\frac{2b^2}{a} = 2ae$

$\Rightarrow b^2 = a^2 e$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{a^2 e}{a^2}}$

$\Rightarrow \mathrm{e}=\sqrt{1-\mathrm{e}}$

On squaring both sides, we get:

$\mathrm{e}^2\mathrm{e}-1=0$

$\Rightarrow \mathrm{e}=\frac{-1\pm\sqrt{1+4}}{2}$

$\Rightarrow \mathrm{e}=\frac{\sqrt{5}-1}{2}$ $(\because \mathrm{e\ cannot\ be\ negative})$

36. Choose the correct answer.

The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is:

(A) 4 (B) 3 (C) 8 (D) $\frac{4}{\sqrt{3}}$

Ans. :

d. $\frac{4}{\sqrt{3}}$

Solution:

$3\mathrm{x}^2+\mathrm{y}^2=12$

$\Rightarrow \frac{\mathrm{x}^2}{4}+\frac{\mathrm{y}^2}{12}=1$

$\therefore\ \mathrm{a}^2=4$

$\Rightarrow \mathrm{a}=2$

and $\mathrm{b}^2=12$

$\Rightarrow \mathrm{b}=2\sqrt{3}$

Since b > a, length of latus rectum $=\frac{2\mathrm{a}^2}{\mathrm{b}}=\frac{2\times4}{2\sqrt{3}}=\frac{4}{\sqrt{3}}$

37. Find the area of $x^2 + y^2 = 49$:

(A) 154 (B) 49 (C) 88 (D) None

Ans. :

a. 154

Solution:

Th eequation $x^2 + y^2 = 49$ describes a circle with 7 as radius So the area is given as $\pi\mathrm{r}^2$

$=\frac{22}{7}\times7^2=154$

38. The equation of the conic $9x^2 - 16y^2 = 144$ is

(A) $\frac{5}{4}$ (B) $\frac{4}{3}$ (C) $\frac{4}{5}$ (D) $\sqrt{7}$

Ans. :

a. $\frac{5}{4}$

Solution:

Standard form of a hyperbola $=\frac{\mathrm{x}^2}{16}-\frac{\mathrm{y}^2}{9}=1$

Here, $a^2 = 16$ and $y^2 = 9$

The eccentricity is calculated in the following way:

$b^2 = a^2(e^2 - 1)$

$\Rightarrow 9 = 16(e^2 - 1)$

$\Rightarrow e^2 - 1 = \frac{9}{16}$

$\Rightarrow e^2 = \frac{25}{16}$

$\Rightarrow e = \frac{5}{4}$

39. The length of latus rectum of the parabola $(x - 2a)^2 + y^2 = x^2$ is:

(A) 2a (B) 3a (C) 6a (D) 4a

Ans. :

d. 4a

Solution:

We have, $(x - 2a)^2 + y^2 = x^2$

$x^2 - 4ax + 4a^2 + y^2 = x^2$

$y^2 = 4ax - 4a^2 = 4a(x - a)$

Comparing it with standard parabola $Y^2 = 4bX$

Y = y, X = x - a, b = a

We know length of latus rectum of parabola $Y^2 = 4bX$ is 4b

length of latus rectum of given parabola is = 4 × a = 4a

40. The locus of a planet orbiting around the sun is:

(A) A circle (B) A straight line (C) A semicircle (D) An ellipse

Ans. :

d. An ellipse

Solution:

It is a fact & proof of it can be seen from higher education physics books.

41. The diameter of a circle described by $9x^2 + 9y^2 = 16$ is:

(A) $\frac{16}{9}$ (B) $\frac{4}{3}$ (C) 4 (D) $\frac{8}{3}$

Ans. :

d. $\frac{8}{3}$

Solution:

Equation of circle is $9x^2 + 9y^2 = 16$

$\Rightarrow x^2 + y^2 = \frac{16}{9}$

$\Rightarrow x^2 + y^2 = (\frac{4}{3})^2$

$\therefore$ Radius of the circle $= \frac{4}{3}$

$\therefore$ Diameter of the circle $= \frac{4\times 2}{3} = \frac{8}{3}$

42. The equation of the circle passing through the origin which cuts off intercept of length 6 and 8 from the axes is:

(A) $x^2 + y^2 - 12x - 16y = 0$ (B) $x^2 + y^2 + 12x + 16y = 0$

(C) $x^2 + y^2 + 6x + 8y = 0$ (D) $x^2 + y^2 - 6x - 8y = 0$

Ans. :

d. $x^2 + y^2 - 6x - 8y = 0$

Solution:

The centre of the required circle is $\left(\frac{6}{2}, \frac{8}{2}\right) = (3, 4)$.

The radius of the required circle is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Hence, the equation of the circle is as follows:

$(x - 3)^2 + (y - 4)^2 = 52$

$\Rightarrow x^2 + y^2 - 6x - 8y = 0$

43. If the length of the tangent from the origin to the circle centered at (2, 3) is 2 then the equation of the circle is:

(A) $(x + 2)^2 + (y - 3)^2 = 3^2$ (B) $(x - 2)^2 + (y + 3)^2 = 3^2$

(C) $(x - 2)^2 + (y - 3)^2 = 3^2$ (D) $(x + 2)^2 + (y + 3)^2 = 3^2$

Ans. :

c. $(x - 2)^2 + (y - 3)^2 = 3^2$

Solution:

Radius of the circle $= \sqrt{(2-0)^2 + (3-0)^2 - 2^2}$

$= \sqrt{(4 + 9 - 4)}$

$= \sqrt{9}$

= 3

So, the equation of the circle = $(x - 2)^2 + (y - 3)^2 = 3^2$

44. The equation of the circle drawn with the two foci of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ end-point of a diameter is

(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 + y^2 = a^2$ (C) $x^2 + y^2 = 2a^2$ (D) $x^2 + y^2 = a^2 - b^2$

Ans. :

d. $x^2 + y^2 = a^2 - b^2$

solution:

We have $r = ae$

Let the equation of the circle be $x^2 + y^2 = r^2$.

Now, $x^2 + y^2 = a^2e^2 \ (\because r = ae)$

$\Rightarrow x^2 + y^2 = a^2\left(1 - \frac{b^2}{a^2}\right) \ \left(\because e = \sqrt{1 - \frac{b^2}{a^2}}\right)$

$\Rightarrow x^2 + y^2 = a^2 - b^2$

$\therefore$ The required equation of the circle $x^2 + y^2 = a^2 - b^2$.

45. Find the Center of circle $x^2 + y^2 - 4x - 8x + 25 = 0$:

(A) (2, 4) (B) (-2, -4) (C) (4, 2) (D) (-4, -2)

Ans. :

a. (2, 4)

Solution:

The general equation of center of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is (-g, -f)

So, the center of circle $x^2 + y^2 - 4x, -8x + 25 = 0$ is (2, 4)

46. The equation $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$ represents

(A) A circle (B) A parabola (C) An ellipse (D) A hyperbola

Ans. :

b. a parabola

Solution:

Comparing the given equation with $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, we get:

a = 16, b = 1, h = 4

We have: $h^2 = 16 = ab$

Thus, the given equation represents a parabola.

47. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is:

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) 4

Ans. :

b. $\frac{4}{3}$

Solution:

Since, the parabola $y^2 = 4ax$ passes through the point (3, 2)

$\Rightarrow 2^2 = 4a \times 3$

$\Rightarrow 4 = 12a$

$\Rightarrow a = \frac{4}{12}$

$\Rightarrow a = \frac{1}{3}$

So, the length of latusrectum $= 4a = 4 \times (\frac{1}{3}) = \frac{4}{3}$

48. If the equation $(4a - 3)x^2 + ay^2 + 6x - 2y + 2 = 0$ represents a circle, then its centre is:

a. (3, -1)
b. (3, 1)
c. (-3, 1)
d. None of these

Ans. :

c. (-3, 1)

Solution:

If the equation (4a - 3) x^2 + ay^2 + 6x - 2y + 2 = 0 represents a circle, then we have:

Coefficient of x^2 = Coefficient of y^2

⇒ 4a - 3 = a

⇒ a = 1

∴ Equation of the circle = x^2 + y^2 + 6x - 2y + 2 = 0

Thus, the coordinates of the centre is (-3, 1).

49. The radius of the circle represented by the equation $3\mathrm{x}^2 + 3\mathrm{y}^2 + (\lambda - 6)\mathrm{y} + 3 = 0$ is:

a. $\frac{3}{2}$

b. $\frac{\sqrt{17}}{2}$

c. $\frac{2}{3}$

d. None of these

Ans. :

a. $\frac{3}{2}$

Solution:

The equation of the circle is $3\mathrm{x}^2 + 3\mathrm{y}^2 + (\lambda - 6)\mathrm{y} + 3 = 0$

∴ Coefficient of $\mathrm{xy} = 0$

$\Rightarrow \lambda = 0$

$\therefore 3\mathrm{x}^2 + 3\mathrm{y}^2 + 9\mathrm{x} - 6\mathrm{y} + 3 = 0$

$\Rightarrow \mathrm{x}^2 + \mathrm{y}^2 + 3\mathrm{x} - 2\mathrm{y} + 1 = 0$

Therefore, the radius of the circle is $\sqrt{\left(\frac{3}{2}\right)^2 + (-1)^2 - 1} = \frac{3}{2}$.

50. If $2\mathrm{x}^2 + \lambda\mathrm{xy} + 2\mathrm{y}^2(\lambda - 4)\mathrm{x} + 6\mathrm{y} - 5 = 0$ is the equation of a circle, then its radius is:

a. $3\sqrt{2}$

b. $2\sqrt{3}$

c. $2\sqrt{2}$

d. None of these

Ans. :

d. None of these

Solution:

The given equation is $2\mathrm{x}^2 + \lambda\mathrm{xy} + 2\mathrm{y}^2 + (\lambda - 4)\mathrm{x} + 6\mathrm{y} - 5 = 0$ which can be rewritten as

$\mathrm{x}^2 + \frac{\lambda\mathrm{xy}}{2} + \mathrm{y}^2 + \frac{(\lambda-4)}{2}\mathrm{x} + 3\mathrm{y} - \frac{5}{2} = 0.$

Comparing the given equation $x^2 + y62 + 2gx + 2fy + c = 0$ with we get: $\lambda = 0$

$\therefore x^2 + y^2 - 2x + 3y - \frac{5}{2} = 0$

$\therefore$ Radius $= \sqrt{(-1)^2 + \left(\frac{3}{2}\right)^2 + \frac{5}{2}} = \sqrt{1 + \frac{9}{4} + \frac{5}{2}} = \sqrt{\frac{23}{4}} = \frac{\sqrt{23}}{2}$

51. The number of integral values of λ for which the equation $x^2 + y^2 + \lambda + (1-\lambda)y + 5 = 0$ is the equation of a circle whose radius cannot exceed 5, is:
    a. 14
    b. 18
    c. 16
    d. None of these

Ans. :

c. 16

Solution:

$\sqrt{\left(\frac{-\lambda}{2}^2\right) + \left(\frac{\lambda-1}{2}\right)^2 - 5} \leq 5$

$\Rightarrow \left(\frac{-\lambda}{2}^2\right) + \left(\frac{\lambda-1}{2}\right) \leq 30$

$\lambda^2 + (\lambda - 1)^2 \leq 120$

$\Rightarrow 2\lambda^2 - 2\lambda - 199 \leq 0$

Using quadratic formula:

$\Rightarrow \lambda = \frac{2 \pm \sqrt{2^2 - 4(2)(-119)}}{2(2)}$

$\Rightarrow \lambda = \frac{2 \pm \sqrt{956}}{4}$

$\Rightarrow \lambda = \frac{1 \pm \sqrt{239}}{2}$

$\Rightarrow \lambda = -7.23,\ 8.23$

$\Rightarrow -7.23 \leq \lambda \leq 8.23$

$\Rightarrow \lambda = -7,\ -6,\ -5,\ -4,\ -3,\ -2,\ -1$ $\quad$ $(\text{if } \lambda \in Z)$

$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,$

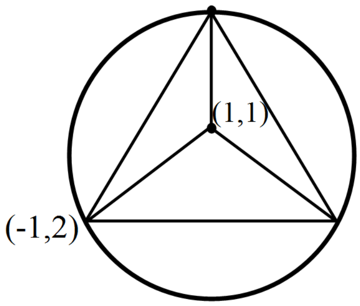
Thus, the number of integral values of λ is 16.

52. If the centroid of an equilateral triangle is (1, 1) and its one vertex is (-1, 2), then the equation of its circumcircle is:
    a. $x^2 + y^2 - 2x - 2y - 3 = 0$
    b. $x^2 + y^2 + 2x - 2y - 3 = 0$
    c. $x^2 + y^2 + 2x + 2y - 3 = 0$
    d. None of these

Ans. :

a. $x^2 + y^2 - 2x - 2y - 3 = 0$

Solution:



The centre of the circumcircle is (1, 1).

Radius of the circumcircle

$\therefore$ Equation of the circle: $= \sqrt{(1+1)^2 + (1-2)^2} = \sqrt{5}$

$(x - 1)^2 + (y - 1)^2 = 5$

$\Rightarrow x^2 + y2 - 2x - 2y - 3 = 0$

53. The vertex of the parabola $(y + a)^2 = 8a (x - a)$ is

a. (-a, -a)

b. (a, -a)

c. (-a, a)

d. None of these

Ans. :

b. (a, a)

Solution:

Given:

The equation of the parabola is $(y + a)^2 = 8a (x - a)$.

Putting X = x - a, Y = y + a

$Y^2 = 8aX$

Vertex = (X = 0, Y = 0) = (x - a = 0, y + a = 0) = (x = a, y = -a)

Hence, the vertex is at (a, a).

54. The equation of the parabola whose vertex is (a, 0) and the directrix has the equation x + y = 3a, is

a. $x^2 + y^2 + 2xy + 6ax + 10ay + 7a^2 = 0$

b. $x^2 - 2xy + y^2 + 6ax + 10ay - 7a^2 = 0$

c. $x^2 - 2xy + y^2 - 6ax + 10ay - 7a^2 = 0$

d. None of these

Ans. :

b. $x^2 - 2xy + y^2 + 6ax + 10ay - 7a^2 = 0$

Solution:

Given:

The vertex is at (a, 0) and the directrix is the line x + y = 3a.

The slope of the line perpendicular to x + y = 3a is 1.

The axis of the parabola is perpendicular to the directrix and passes through the vertex.

$\therefore$ Equation of the axis of the parabola = y − 0 = 1(x - a) ...(1)

Intersection point of the directrix and the axis is the intersection point of (1) and x + y = 3a.

Let the intersection point be K.

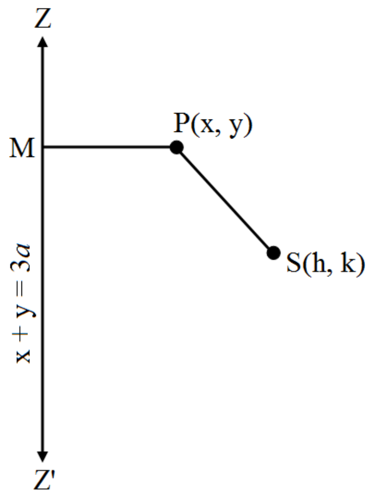
Therefore, the coordinates of K are (2a, a)

The vertex is the mid-point of the segment joining K and the focus (h, k).

$\therefore\ \mathrm{a} = \frac{2\mathrm{a}+\mathrm{h}}{2},\ 0 = \frac{\mathrm{a}+\mathrm{k}}{2}$

h = 0, k = -a

Let P (x, y) be any point on the parabola whose focus is S (h, k) and the directrix is x + y= 3a.



Draw PM perpendicular to x + y = 3a.

Then, we have:

SP = PM

$\Rightarrow$ SP^2 = PM^2

$\Rightarrow\ (\mathrm{x}-0)^2 + (\mathrm{y}+\mathrm{a})^2 = \left(\frac{\mathrm{x}+\mathrm{y}-3\mathrm{a}}{\sqrt{2}}\right)^2$

$\Rightarrow\ \mathrm{x}^2 + (\mathrm{y}+\mathrm{a})^2 = \left(\frac{\mathrm{x}+\mathrm{y}-3\mathrm{a}}{\sqrt{2}}\right)^2$

$\Rightarrow\ 2\mathrm{x}^2 + 2\mathrm{y}^2 + 2\mathrm{a}^2 + 4\mathrm{ay} = \mathrm{x}^2 + \mathrm{y}^2 + 9\mathrm{a}^2 + 2\mathrm{xy} - 6\mathrm{ax} - 6\mathrm{ay}$

$\Rightarrow\ \mathrm{x}^2 + \mathrm{y}^2 - 7\mathrm{a}^2 + 10\mathrm{ay} + 6\mathrm{ax} = 0$

55. The eccentricity of the conic $9\mathrm{x}^2 + 25\mathrm{y}^2 = 225$ is:

a. $\frac{2}{5}$

b. $\frac{4}{5}$

c. $\frac{1}{3}$

d. $\frac{1}{5}$

e. $\frac{3}{5}$

Ans. :

b. $\frac{4}{5}$

$9x^2 + 25y^2 = 225$

Solution:

$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$

Comparing it with $\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ we get:

$a = 5$ and $b = 3$

Here, a > b, so the major and the minor axes of the ellipse are along the x−axis and y−axis, respectively.

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{9}{25}}$

$\Rightarrow e = \sqrt{\frac{16}{25}}$

$\Rightarrow e = \frac{4}{5}$

56. The eccentricity of the ellipse, if the distance between the foci is equal to the length of the latus-rectum, is:

a. $\frac{\sqrt{5}-1}{2}$

b. $\frac{\sqrt{5}+1}{2}$

c. $\frac{\sqrt{5}-1}{4}$

d. none of these

Ans. :

a. $e = \frac{\sqrt{5}-1}{2}$

Solution:

According to the question, the distance between the foci is equal to the length of the latus rectum.

$\frac{2b^2}{a} = 2ae$

$\Rightarrow b^2 = a^2e$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{a^2e}{a^2}}$

$\Rightarrow e = \sqrt{1 - e}$

On squaring both sides, we get:

$e^2e - 1 = 0$

$\Rightarrow e = \frac{-1 \pm \sqrt{1+4}}{2}$

$\Rightarrow e = \frac{\sqrt{5}-1}{2} \quad (\because \text{e cannot be negative})$

57. The difference between the lengths of the major axis and the latus-rectum of an ellipse is

a. ae

b. 2ae

c. ae^2

d. $2ae^2$

Ans. :

d. $2ae^2$

Solution:

Length of the latus rectum $= \frac{2b^2}{a}$

and $e = \sqrt{1 - \frac{b^2}{a^2}}$

$a^2e^2 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - a^2e^2$

$\Rightarrow b^2 = a^2(1 - e^2)$

$\therefore$ Length of the latus rectum $= \frac{2a^2(1-e^2)}{a} = 2a(1 - e^2)$

Length of the major axis = 2a

Difference between length of latus rectum and length of major axis

$= 2a - 2a(1 - e^2)$

$= 2a - 2a + 2ae^2$

$= 2ae^2$

58. A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation PA − PB = k ($k \neq 0$), then the locus of P is

a. A hyperbola.

b. A branch of the hyperbola.

c. A parabola.

d. An ellipse.

Ans. :

a. A hyperbola.

Solution:

Let P(x, y) be any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

By definition, we have:

$PA = e\left(x - \frac{a}{e}\right) = ex - a$

and $PB = e\left(x + \frac{a}{e}\right) = ex + a$

$\therefore PB - PA = (ex+a) - (ex - a) = 2a = k$

59. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2 \alpha + y^2 = 25$, then α =

a. $\frac{\pi}{6}$
b. $\frac{\pi}{4}$
c. $\frac{\pi}{3}$
d. $\frac{\pi}{2}$

Ans. :

b. $\frac{\pi}{4}$

Solution:

The hyperbola $x^2 - y^2 \sec^2 \alpha = 5$ can be rewritten in the following way:

$\frac{x^2}{5} - \frac{y^2}{5\cos^2 a} = 1$

This is the standard form of a hyperbola, where $a^2 = 5$ and $b^2 = 5\cos^2 \alpha$.

$\Rightarrow b^2 = a^2(e_1^2 - 1)$

$\Rightarrow 5\cos^2 \alpha = 5(e_1^2 - 1)$

$\Rightarrow e_1^2 = \cos^2 \alpha + 1...(1)$

The ellipse $x^2 \sec^2 \alpha + y^2 = 25$ can be rewritten in the following way:

$\frac{x^2}{25\cos^2 \alpha} + \frac{y^2}{25} = 1$

This is the standard form of an ellipse, where $a^2 = 25$ and $b^2 = 25\cos^2 \alpha$

$b^2 = a^2(1 - e_2^2)$

$\Rightarrow e_2^2 = 1 - \cos^2 \alpha$

$\Rightarrow e_2^2 = \sin^2 \alpha \ldots (2)$

According to the question,

$\cos^2 \alpha + 1 = 3(\sin^2 \alpha)$

$\Rightarrow 2 = 4\sin^2 \alpha$

$\Rightarrow \sin \alpha = \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha = \frac{\pi}{4}$

60. The distance between the directrices of the hyperbola $x = 8\sec\theta, y = 8,$ is

a. $8\sqrt{2}$
b. $16\sqrt{2}$
c. $4\sqrt{2}$
d. $6\sqrt{2}$

Ans. :

a. $8\sqrt{2}$

Solution:

We have:

$x = 8\sec\theta, y = 8\tan\theta$

On squaring and subtracting:

$x^2 - y^2 = 8\sec^2\theta - 8\tan^2\theta$

$\Rightarrow x^2 - y^2 = 8$

$\Rightarrow \frac{x^2}{8} - \frac{y^2}{8} = 1$

$\therefore a = b = c$

Distance between the directrices of the hyperbola $= \frac{2a^2}{\sqrt{a^2+b^2}}$

Distance between the directrices $= \frac{2\times 64}{\sqrt{64+64}}$

$= \frac{128}{8\sqrt{2}}$

$= \frac{16}{\sqrt{2}}$

$= 8\sqrt{2}$

61. If the tangent to the circle $x^2 + y^2 = r^2$ at the point (a,b) meets the coordinate axes at the point A and B, and O is the origin, then the area of the triangle OAB is

(A) $\frac{r^4}{2ab}$ (B) $\frac{r^4}{ab}$ (C) $\frac{r^2}{2ab}$ (D) $\frac{r^2}{ab}$

Ans. : a

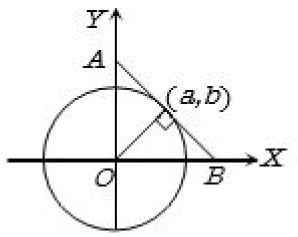
(a) Obviously $r = \sqrt{a^2 + b^2}$

Equation of AB is $ax + by = r^2$ or $\frac{x}{r^2/a} + \frac{y}{r^2/b} = 1$

$\Rightarrow OA = \frac{r^2}{a}$ and $OB = \frac{r^2}{b}$

Hence the area is $\frac{1}{2} \cdot \frac{r^2}{a} \cdot \frac{r^2}{b}$

$= \frac{1}{2} \frac{r^4}{ab}$.



62. If the line $3x + 4y - 1 = 0$ touches the circle $(x-1)^2 + (y-2)^2 = r^2$, then the value of r will be

(A) 2 (B) 5 (C) $\frac{12}{5}$ (D) $\frac{2}{5}$

Ans. : a

(a) If the line $3x + 4y - 1 = 0$ touches the circle $(x-1)^2 + (y-2)^2 = r^2$, then the perpendicular from centre of circle on line is equal to the radius of circle.

$i.e.,$ $\left|\frac{3+8-1}{5}\right| = r$ or $r = 2$.

63. If $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ touches the circle $x^2 + y^2 = a^2$, then point $(1/\alpha, 1/\beta)$ lies on a/an

(A) Straight line (B) Circle (C) Parabola (D) Ellipse

Ans. : b

(b) $y = -\frac{\beta}{\alpha}x + \beta$ touches the circle,

$\beta^2 = a^2 \left(1 + \frac{\beta^2}{\alpha^2}\right)$

==> $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$=\frac{1}{a^2}$

Locus of $\left(\frac{1}{\alpha},\frac{1}{\beta}\right)$ is x^2+y^2

$=\left(\frac{1}{a}\right)^2$.

64. The length of the tangent from the point $(4,5)$ to the circle $x^2+y^2+2x-6y=6$ is

(A) $\sqrt{13}$ (B) $\sqrt{38}$ (C) $2\sqrt{2}$ (D) $2\sqrt{13}$

Ans. : a

(a) Length of the tangent

$=\sqrt{4^2+5^2+2\cdot 1\cdot 4+2(-3)\cdot 5-6}$

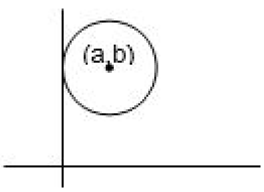
$=\sqrt{13}$.

65. Tangents drawn from origin to the circle $x^2+y^2-2ax-2by+b^2=0$ are perpendicular to each other, if

(A) $a-b=1$ (B) $a+b=1$ (C) $a^2=b^2$ (D) $a^2+b^2=1$

Ans. : c

(c) Obviously, tangents from origin will be perpendicular if it touches both axes $i.e.$, $a=b$ or $a^2=b^2$.



66. The equation of the tangents to the circle $x^2+y^2+4x-4y+4=0$ which make equal intercepts on the positive coordinate axes is given by

(A) $x+y+2\sqrt{2}=0$ (B) $x+y=2\sqrt{2}$ (C) $x+y=2$ (D) None of these

Ans. : b

(b) Equation of tangent is of the form $x+y+c=0$ and also it obeys condition of tangency,

$i.e.$, $\left|\frac{-2+2+c}{\sqrt{2}}\right|=\sqrt{4+4-4}$

$\Rightarrow c=\pm 2\sqrt{2}$

But for positive intercepts, $c=-2\sqrt{2}$

The tangent is $x+y=2\sqrt{2}$.

67. The gradient of the tangent line at the point $(a\cos\alpha, a\sin\alpha)$ to the circle $x^2+y^2=a^2$, is

(A) $\tan\alpha$ (B) $\tan(\pi-\alpha)$ (C) $\cot\alpha$ (D) $-\cot\alpha$

Ans. : d

(d) Equation of a tangent at $(a\cos\alpha,\, a\sin\alpha)$ to the circle $x^2+y^2=a^2$ is $ax\cos\alpha+ay\sin\alpha=a^2$.

Hence its gradient is $-\frac{a\cos\alpha}{a\sin\alpha}=-\cot\alpha$.

68. $y-x+3=0$ is the equation of normal at $\left(3+\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ to which of the following circles

(A) $\left(x-3-\frac{3}{\sqrt{2}}\right)^2+\left(y-\frac{\sqrt{3}}{2}\right)^2=9$
(B) $\left(x-3-\frac{3}{\sqrt{2}}\right)^2+y^2=6$
(C) $(x-3)^2+y^2=9$
(D) $(x-3)^2+(y-3)^2=9$

Ans. : c

(c) Trick : Only option (c) satisfies the given co-ordinates of the point.

69. Which of the following lines is a tangent to the circle $x^2+y^2=25$ for all values of m.....

(A) $y=mx+25\sqrt{1+m^2}$
(B) $y=mx+5\sqrt{1+m^2}$
(C) $y=mx+25\sqrt{1-m^2}$
(D) $y=mx+5\sqrt{1-m^2}$

Ans. : b

(b) Line $y=mx+c$ is a tangent if

$c=\pm a\sqrt{1+m^2}$

$\therefore y=mx+5\sqrt{1+m^2}$.

70. Line $y=x+a\sqrt{2}$ is a tangent to the circle $x^2+y^2=a^2$ at

(A) $\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$
(B) $\left(-\frac{a}{\sqrt{2}},-\frac{a}{\sqrt{2}}\right)$
(C) $\left(\frac{a}{\sqrt{2}},-\frac{a}{\sqrt{2}}\right)$
(D) $\left(-\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$

Ans. : d

(d) Suppose that the point be (h,k).

Tangent at (h,k) is $hx+ky=a^2\equiv x-y=-\sqrt{2}a$

or $\frac{h}{1}=\frac{k}{-1}=\frac{a^2}{-\sqrt{2}a}$

or $h=-\frac{a}{\sqrt{2}},\ k=\frac{a}{\sqrt{2}}$

Therefore, point of contact is $\left(-\frac{a}{\sqrt{2}},\ \frac{a}{\sqrt{2}}\right)$.

71. The equations of the tangents to the circle $x^2+y^2=13$ at the points whose abscissa is 2, are

(A) $2x+3y=13,\, 2x-3y=13$
(B) $3x+2y=13,\, 2x-3y=13$
(C) $2x+3y=13,\ 3x-2y=13$
(D) None of these

Ans. : a

(a) Let the point be $(2,y')$, then

$2^2+y'^2=13$

$\Rightarrow y'=\pm 3$

Hence the required tangents are $2x \pm 3y = 13$.

72. If a line passing through origin touches the circle $(x-4)^2+(y+5)^2=25$, then its slope should be

(A) $\pm\frac{3}{4}$ (B) 0 (C) ± 3 (D) ± 1

Ans. : b

(b) Let equation of line be $y = mx$ or $y - mx = 0$

Then applying condition for tangency,

$\left|\frac{-5-4m}{\sqrt{1+m^2}}\right| = 5$

$\Rightarrow 25+16m^2+40m = 25+25m^2$

$\Rightarrow 9m^2-40m=0 \Rightarrow m=0$ or $m=\frac{40}{9}$.

73. If the line $y\cos\alpha = x\sin\alpha + a\cos\alpha$ be a tangent to the circle $x^2+y^2=a^2$, then

(A) $\sin^2\alpha = 1$ (B) $\cos^2\alpha = 1$ (C) $\sin^2\alpha = a^2$ (D) $\cos^2\alpha = a^2$

Ans. : b

(b) The tangent is $y\cos\alpha = x\sin\alpha + a\cos\alpha$

$\Rightarrow y = x\tan\alpha + a$

It is a tangent to the circle $x^2+y^2=a^2$, if

$a^2 = a^2(1+\tan^2\alpha)$

$\Rightarrow \sec^2\alpha = 1$

$\Rightarrow \cos^2\alpha = 1$.

74. The angle between the tangents to the circle $x^2+y^2=169$ at the points $(5,12)$ and $(12,-5)$ is .... ^{o}

(A) 30 (B) 45 (C) 60 (D) 90

Ans. : d

(d) The equations of tangents to the given circle will be $5x+12y=169$ and $12x-5y=169$.

Obviously the angle between the tangents is 90^o.

75. An infinite number of tangents can be drawn from $(1,2)$ to the circle $x^2+y^2-2x-4y+\lambda=0$, then $\lambda =$

(A) -20 (B) 0

(C) 5 (D) Cannot be determined

Ans. : c

(c) Clearly the point $(1,2)$ is the centre of the given circle and infinite tangents can only be drawn on a point circle.

Hence radius should be 0.

$\therefore \sqrt{1^2+2^2-\lambda} = 0$

$\Rightarrow \lambda = 5$.

76. The line $(x-a)\cos\alpha+(y-b)\ \ \sin\alpha=r$ will be a tangent to the circle $(x-a)^2+(y-b)^2=r^2$

(A) If $\alpha=30^o$

(B) If $\alpha=60^o$

(C) For all values of α

(D) None of these

Ans. : c

(c) According to the condition of tangency

$r=\frac{a\cos\alpha+b\sin\alpha-(a\cos\alpha+b\sin\alpha)-r}{\sqrt{\cos^2\alpha+\sin^2\alpha}}$

$\Rightarrow r=|-r|\ \Rightarrow r=r$.

Therefore, it is a tangent to the circle for all values of α.

77. If the line $lx+my+n=0$ be a tangent to the circle $(x-h)^2+(y-k)^2=a^2,$ then

(A) $hl+km+n=a^2(l^2+m^2)$

(B) $(hl+km+n)^2=a(l^2+m^2)$

(C) $(hl+km+n)^2=a^2(l^2+m^2)$

(D) None of these

Ans. : c

(c) According to the condition, $a=\frac{hl+mk+n}{\sqrt{l^2+m^2}}$

$\Rightarrow (hl+km+n)^2$

$=a^2(l^2+m^2)$.

78. The line $x\cos\alpha+y\sin\alpha=p$ will be a tangent to the circle $x^2+y^2-2ax\cos\alpha-2ay\sin\alpha=0$, if $p=$

(A) 0 or a

(B) 0

(C) $2a$

(D) 0 or $2a$

Ans. : d

(d) $x\cos\alpha+y\sin\alpha-p=0$ is a tangent, if perpendicular from centre on it is equal to radius of the circle.

Here centre is $(a\cos\alpha,\ a\sin\alpha)$ and radius is a.

$\left|\frac{a\cos^2\alpha+a\sin^2\alpha-p}{\sqrt{1}}\right|=a$

$i.e.\ |a-p|\ =a\Rightarrow p=0$ or $p=2a$.

79. The equations of the tangents to the circle $x^2+y^2=36$ which are inclined at an angle of 45^o to the x-axis are

(A) $x+y=\pm\sqrt{6}$

(B) $x=y\pm3\sqrt{2}$

(C) $y=x\pm6\sqrt{2}$

(D) None of these

Ans. : c

(c) $y=mx+c$ is a tangent, if $c=\pm a\sqrt{1+m^2}$, where $m=\tan45^o=1$

$\therefore$ The equation is $y=x\pm6\sqrt{2}$.

80. If the line $y=\sqrt{3}x+k$ touches the circle $x^2+y^2=16$, then $k=$

(A) 0

(B) 2

(C) 4

(D) 8

Ans. : d

(d) $k=\pm 4\sqrt{1+3}=\pm 8$.

81. The equations of the tangents to the circle $x^2+y^2=50$ at the points where the line $x+7=0$ meets it, are

(A) $7x\pm y+50=0$ (B) $7x\pm y-5=0$ (C) $y\pm 7x+5=0$ (D) $y\pm 7x-5=0$

Ans. : a

(a) Points where $x+7=0$ meets the circle $x^2+y^2=50$ are $(-7,\ 1)$ and $(-7,\ -1)$.
Hence equations of tangents at these points are $-7x\pm y=50$ or $7x\pm y+50=0$.

82. If the length of tangent drawn from the point $(5,3)$ to the circle $x^2+y^2+2x+ky+17=0$ be 7, then k =

(A) 4 (B) -4 (C) -6 (D) $\frac{13}{2}$

Ans. : b

(b) According to the condition,

$\sqrt{(5)^2+(3)^2+2(5)+k(3)+17}=7$

$\Rightarrow 61+3k=49 \Rightarrow k=-4$.

83. If the point $(2,0),(0,1),(4,5)$ and $(0,c)$ are con-cyclic, then c is equal to

(A) $-1,\ -\frac{3}{14}$ (B) $-1,\ -\frac{14}{3}$ (C) $\frac{14}{3},\ 1$ (D) None of these

Ans. : c

(c) Circle with $(2,0),(0,1)$ as end points of diameter is $(x-2)x+(y-1)y=0$ and line through these two points is $y-0=\left(\frac{-1}{2}\right)(x-2)$ or $2y+x-2=0$

Family of circles through these two points are

$x(x-2)+y(y-1)+\lambda(2y+x-2)=0$.

It passes through $(4,5)$.

i.e., $4(2)+5(4)+\lambda(10+4-2)=0 \Rightarrow \lambda=\frac{-7}{3}$.

Hence equation of circle is

$x(x-1)+y(y-1)-\frac{7}{3}(2y+x-2)=0$

It passes through $(0,c)$, therefore

$c(c-1)-\frac{7}{3}(2c-2)=0$

$\Rightarrow 3c^2-17c+14=0$ or $c=\frac{14}{3}$ and 1.

84. Area of the circle in which a chord of length $\sqrt{2}$ makes an angle $\frac{\pi}{2}$ at the centre is

(A) $\frac{\pi}{2}$ (B) 2π (C) π (D) $\frac{\pi}{4}$

Ans. : c

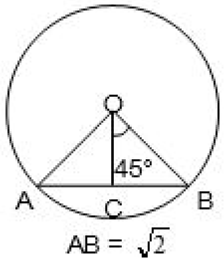
(c) Let AB be the chord of length $\sqrt{2}$,
O be centre of the circle and let OC be the perpendicular from O on AB.

Then $AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

In $\Delta\, OBC,\ OB = BC\,\text{cosec}45^o$

$= \frac{1}{\sqrt{2}}.\sqrt{2} = 1$

$\therefore$Area of the circle $= \pi(OB)^2 = \pi$.



85. The equation of the circle whose diameter lies on $2x + 3y = 3$ and $16x - y = 4$ which passes through $(4,6)$ is

(A) $5(x^2 + y^2) - 3x - 8y = 200$

(B) $x^2 + y^2 - 4x - 8y = 200$

(C) $5(x^2 + y^2) - 4x = 200$

(D) $x^2 + y^2 = 40$

Ans. : a

(a) Let point $(x_1,\ y_1)$ on the diameter.

$\Rightarrow 2x_1 + 3y_1 = 3 \,....(i)$

$16x_1 - y_1 = 4\,....(ii)$

On solving (i) and (ii), we get centre,

$\Rightarrow x_1 = \frac{3}{10},\ y_1 = \frac{4}{5}$

$\therefore$ Equation of circle,

$(x - x_1)^2 + (y - y_1)^2 = r^2$

$\Rightarrow \left(x - \frac{3}{10}\right)^2 + \left(y - \frac{4}{5}\right)^2 = r^2$

Circle passes through $(4,6)$.

So, $r^2 = \left(\frac{37}{10}\right)^2 + \left(\frac{26}{5}\right)^2$

$\Rightarrow r^2 = \frac{4073}{100}$

$\therefore$ Required equation of circle is

$\left(x - \frac{3}{10}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{4073}{100}$

$\Rightarrow 5(x^2 + y^2) - 3x - 8y = 200$.

86. The equation of the circle with centre at $(1,-2)$ and passing through the centre of the given circle $x^2 + y^2 + 2y - 3 = 0$, is

(A) $x^2 + y^2 - 2x + 4y + 3 = 0$

(B) $x^2 + y^2 - 2x + 4y - 3 = 0$

(C) $x^2 + y^2 + 2x - 4y - 3 = 0$

(D) $x^2 + y^2 + 2x - 4y + 3 = 0$

Ans. : a

(a) According to the question, the required circle passes through $(0,-1)$.

Therefore, the radius is the distance between the points $(0,-1)$ and $(1,-2)$ i.e., $\sqrt{2}$.

Hence the equation is $(x-1)^2+(y+2)^2=(\sqrt{2})^2$

$\Rightarrow x^2+y^2-2x+4y+3=0$

Trick : Since this is the only circle passing through $(0,-1)$.

87. The equation of the circle which passes through the points $(2,3)$ and $(4,5)$ and the centre lies on the straight line $y-4x+3=0$, is

(A) $x^2+y^2+4x-10y+25=0$ (B) $x^2+y^2-4x-10y+25=0$

(C) $x^2+y^2-4x-10y+16=0$ (D) $x^2+y^2-14y+8=0$

Ans. : b

(b) First find the centre. Let centre be (h,k), then

$\sqrt{(h-2)^2+(k-3)^2}=\sqrt{(h-4)^2+(k-5)^2}$ ....(i)

and $k-4h+3=0$ ....(ii)

From (i), we get $-4h-6k+8h+10k=16+25-4-9$

or $4h+4k-28=0$ or $h+k-7=0$ ....(iii)

From (iii) and (ii), we get (h,k) as $(2,5)$.

Hence centre is $(2,5)$ and radius is 2.

Now find the equation of circle.

Trick : Obviously, circle $x^2+y^2-4x-10y+25=0$ passes through $(2,3)$ and $(4,5)$.

88. The equation of the circle passing through the origin and cutting intercepts of length 3 and 4 units from the positive axes, is

(A) $x^2+y^2+6x+8y+1=0$ (B) $x^2+y^2-6x-8y=0$

(C) $x^2+y^2+3x+4y=0$ (D) $x^2+y^2-3x-4y=0$

Ans. : d

(d) Obviously the centre of the circle is $\left(\frac{3}{2},\,2\right)$.

Therefore, the equation of circle is

$\left(x-\frac{3}{2}\right)^2+(y-2)^2=\left(\frac{5}{2}\right)^2$

$\Rightarrow x^2+y^2-3x-4y=0$.

89. The equation of a circle which touches both axes and the line $3x-4y+8=0$ and whose centre lies in the third quadrant is

(A) $x^2+y^2-4x+4y-4=0$ (B) $x^2+y^2-4x+4y+4=0$

(C) $x^2+y^2+4x+4y+4=0$ (D) $x^2+y^2-4x-4y-4=0$

Ans. : c

(c) The equation of circle in third quadrant touching the coordinate axes with centre $(-a,\,-a)$ and radius 'a' is

$x^2+y^2+2ax+2ay+a^2=0$ and we know $\left|\frac{3(-a)-4(-a)+8}{\sqrt{9+16}}\right|=a\Rightarrow a=2$

Hence the required equation is $x^2+y^2+4x+4y+4=0$.

Trick : Obviously the centre of the circle lies in III quadrant, which is given by (c).

90. Consider two curves $C_1 : y^2 = 2x$ and $C_2 : x^2 + y^2 - 3x + 2 = 0$, then

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

Ans. : (D) C_1 and C_2 neither intersect nor touch each other

91. If (x,y) is a variable point on the curve $x^2 + y^2 - 2x - 2y - 2 = 0$, then minimum value of the expression $\frac{8}{(x-1)^2} - \frac{(y-1)^2}{4}$ is equal to

(A) -2 (B) -1 (C) 1 (D) 2

Ans. : d

$x^2 + y^2 - 2x - 2y - 2 = 0$

centre $\equiv (1,1)$, radius $= 2$

co-ordinate of a point on the curve

$\equiv (1 + 2\cos\theta, 1 + 2\sin\theta)$

$\Rightarrow \frac{8}{(x-1)^2} + \frac{(y-1)^2}{4} = \frac{2}{\cos^2\theta} + \sin^2\theta$

using $A.M. \geq G.M$

min. value $= 2$ at $\sin\theta = 1$

92. The locus of the centre of the circle $\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0$ is :-

(A) $x^2 - y^2 = 1$ (B) $x^2 + y^2 = 1$ (C) $y^2 = x^2$ (D) $x^2 + y^2 = 2$

Ans. : b

$x = -cos\theta$

$y = -sin\theta$

$x^2 + y^2 = 1$

93. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is

(A) $x + y = 2$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x + y = 1$

Ans. : c

Let mid point be (h,k), then equation of chord is $hx + ky = h^2 + k^2$ Now on homoginising $x^2 + y^2 = 4$ with the help of this chord

$x^2 + y^2 = 4\left(\frac{hx+ky}{h^2+k^2}\right)^2$

Now as subtended angle is $90^\circ \Rightarrow$ Coeff. of $x^2 +$ Coeff. of $y^2 = 0$

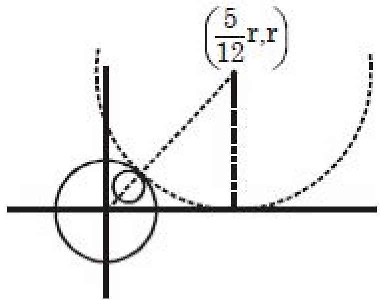
$1 - \frac{4h^2}{(h^2+k^2)^2} + 1 - \frac{4k^2}{(h^2+k^2)^2} = 0,$ represents a pair of straight lines passing through origin and intersection points of circle and chords.

$2\left(x^2+y^2\right)^2=4\left(x^2+y^2\right) \Rightarrow x^2+y^2=2$

94. Let a circle $S=0$ touches both the circles $x^2+y^2=400$ and $x^2+y^2-10x-24y+120=0$ externally and also touches $x-$ axis. The radius of circle $S=0$ is

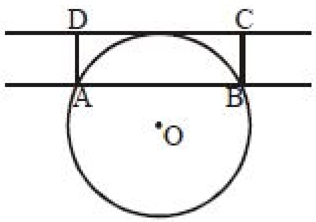
(A) 200 (B) 33 (C) 120 (D) 240

Ans. : d



$$\frac{13}{12}r-r=20 \Rightarrow r=240$$

95. A variable straight line AB divides the circumference of the circle $x^2+y^2=25$ in the ratio $1:2$. If a tangent CD is drawn to the smaller arc parallel to AB , such that $ABCD$ is a rectangle, then locus of C & D is (as shown in the figure)



(A) $x^2+y^2=\frac{175}{4}$ (B) $x^2+y^2=36$ (C) $x^2+y^2=40$ (D) $x^2+y^2=20$

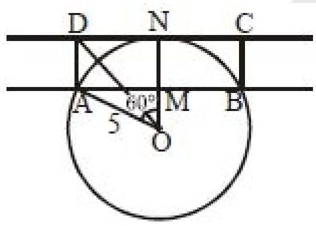
Ans. : a

from figure $AM=DN=5\sin 60^\circ=\frac{5\sqrt{3}}{2}$

Now in $\Delta \mathrm{OND} \Rightarrow (\mathrm{OD})^2=(\mathrm{DN})^2+(\mathrm{ON})^2$

$=\frac{75}{4}+25=\frac{175}{4}$

So locus of C and D is $x^2+y^2=\frac{175}{4}$



96. The centres of a set of circles, each of radius 2, lie on the circle. $x^2+y^2=36$ The locus of any point in the set is -

(A) $4 \leqslant x^2+y^2 \leqslant 16$ (B) $16 \leqslant x^2+y^2 \leqslant 64$

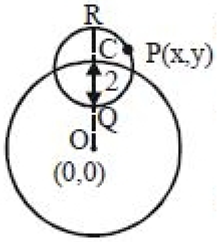
(C) $36 \leqslant x^2+y^2 \leqslant 64$ (D) $16 \leqslant x^2+y^2 \leqslant 36$

Ans. : b

$OQ \leq OP \leq OR$

$4 \leq \sqrt{x^2+y^2} \leq 8$

$16 \leq x^2+y^2 \leq 64$



97. The equation of the locus of the mid points of the chords of the circle $4x^2+4y^2-12x+4y+1=0$ that subtend an angle of $\frac{2\pi}{3}$ at its centre is

(A) $16(x^2+y^2)-48x+16y+31=0$ (B) $16(x^2+y^2)-48x-16y+31=0$

(C) $16(x^2+y^2)+48x+16y+31=0$ (D) $16(x^2+y^2)+48x-16y+31=0$

Ans. : a

For the circle $4x^2+4y^2-12x+4y+1=0$

Radius $=3/2$

Let $E(x,y)$ is mid-point of chord AB So, in ΔOAE

$\frac{3}{2}\cos 60^\circ = \sqrt{\left(\frac{3}{2}-x\right)^2+\left(y+\frac{1}{2}\right)^2}$

$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{2}\right)^2 + x^2 - 3x + y^2 + \left(\frac{1}{2}\right)^2 + y$

$x^2+y^2-3x+y+\frac{10}{4}-\frac{9}{16}=0$

$\Rightarrow x^2+y^2-3x+y+\frac{31}{16}=0$

98. Tangents are drawn to a unit circle with centre at the origin from each point on the line $2x+y=4$. Then the equation to the locus of the middle point of the chord of contact is

(A) $2(x^2+y^2)=x+y$ (B) $2(x^2+y^2)=x+2y$

(C) $4(x^2+y^2)=2x+y$ (D) none

Ans. : c

(x_1,y_1) lies on $2x+y=4$

$\Rightarrow \ 2x_1+y_1=4$....(1)

chord of contact w.r.t. (x_1,y_1)

$xx_1+yy_1=1$

also equation of chord whose mid point is (h,k)

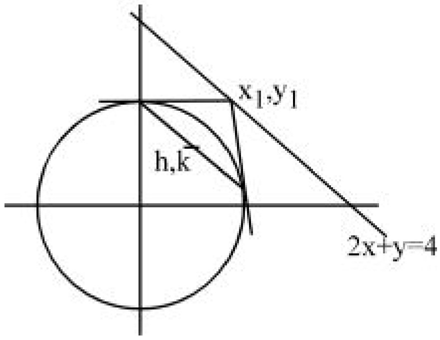
$h^2+k^2=hx+ky$

$\therefore \frac{x_1}{h}=\frac{y_1}{k}=\frac{1}{h^2+k^2} \Rightarrow x_1=\frac{h}{h^2+k^2}$; $y_1=\frac{k}{h^2+k^2}$

substitute in (1)

$2 \cdot \frac{h}{h^2+k^2} + \frac{h}{h^2+k^2} = 4$

locus $= 4(x^2+y^2) = 2x+y$



99. Tangents are drawn to the circle $x^2+y^2=1$ at the points where it is met by the circles, $x^2+y^2-(\lambda+6)x+(8-2\lambda)y-3=0$. λ being the variable . The locus of the point of intersection of these tangents is :

(A) $2x-y+10=0$ (B) $x+2y-10=0$ (C) $x-2y+10=0$ (D) $2x+y-10=0$

Ans. : a

compare chord of contact of the pair of tangents from (x_1, y_1) to the circle $x^2+y^2=1$ with the common chord between the two circles and eliminate λ

Locus of point of intersection of tangents

chord of contact of (x_1, y_1) w.r.t. $x^2+y^2=1$ is $xx_1+yy_1=1 \quad (AB).....(1)$

AB is also common chord between two circles

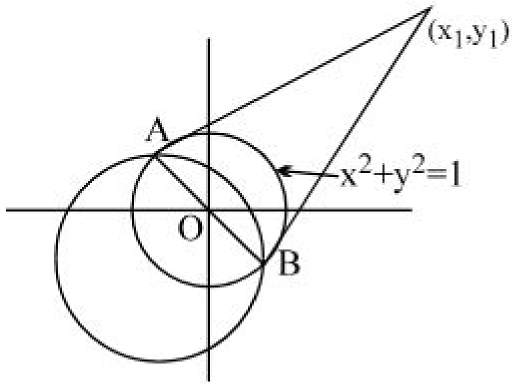
$\therefore \ -1+(\lambda+6)x-(8-2\lambda)y+3=0$

$\Rightarrow (\lambda+6)x-(8-2\lambda)y+2=0.....(2)$

comparing (1) and (2) we get

$\frac{x_1}{\lambda+6} = \frac{y}{2\lambda-8} = \frac{-1}{2}$

eliminate $\lambda \Rightarrow 2x-y+10=0$



100. The locus of the centers of the circles which cut the circles $x^2+y^2+4x-6y+9=0$ and $x^2+y^2-5x+4y-2=0$ orthogonally is

(A) $9x+10y-7=0$ (B) $x-y+2=0$ (C) $9x-10y+11=0$ (D) $9x+10y+7=0$

Ans. : c

Locus of the centre of the ⊙ cutting $S_1=0$ and $S_2=0$ orthogonally is the radical axis between $S_1=0$ and $S_2=0$

Let out circle be $x^2+y^2+2gf+2fy+c=0$

conditions $2(-g)(-2)+2(-f)(3)=c+9$

and $2(-g)(5/2)+2(-f)(-2)=c-2$

$\therefore ag-10\,f=11$

$\therefore$ locus of centre $9x-10y+11=0$

101. The number of direct common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-8x-8y+7=0$, is

(A) 0 (B) 1 (C) 2 (D) 3

Ans. : c

$\mathrm{C}_1(0,0)$ and $\mathrm{r}_1=2$

Also, $C_2(4,4)$ and $r_2=5$

Since, $c_1c_2=4\sqrt{2}$

$\Rightarrow \quad |\mathrm{r}_1-\mathrm{r}_2|<\mathrm{c}_1\mathrm{c}_2<\mathrm{r}_1+\mathrm{r}_2$

$\Rightarrow$ Circles are intersecting.

$\Rightarrow$ Two direct common tangents exist.

102. The circles $x^2+y^2+2x-2y+1=0$ and $x^2+y^2-2x-2y+1=0$ touch each other :-

(A) externally at $(0,1)$ (B) internally at $(0,1)$

(C) externally at $(1,0)$ (D) internally at $(1,0)$

Ans. : a

The centres of the two circles are $\mathrm{C}_1(-1,1)$ and $\mathrm{C}_2(1,1)$ and both have radii equal to 1. We have

$C_1C_2 = 2$ and sum of the radii $= 2$
So, the two circles touch each other externally.
The equation of the common tangent is obtained by subtracting the two equations.
The equation of the common tangnet is
$4x = 0 \Rightarrow x = 0$
Putting $x = 0$ in the equation of the either circle,
we get
$y^2 - 2y + 1 = 0 \Rightarrow (y-1)^2 = 0y = 1$
Hence, the points where the two circles touch is
$(0,1)$

103. The number of integral values of λ for which $x^2 + y^2 + \lambda x + (1-\lambda)y + 5 = 0$ is the equation of a circle whose radius cannot exceed 5 , is

(A) 14 (B) 18 (C) 16 (D) None of these

Ans. : c

Here, radius $\sqrt{\left(\frac{\lambda}{2}\right)^2 + \left(\frac{1-\lambda}{2}\right)^2 - 5 \leq 5}$
$\Rightarrow 2\lambda^2 - 2\lambda - 119 \leq 0$
$\therefore \frac{1-\sqrt{239}}{2} \leq \lambda \leq \frac{1+\sqrt{239}}{2}$
$\Rightarrow -7.2 \leq \lambda \leq 8.2(\text{ nearly })$
$\therefore \lambda = -7, -6, \ldots, 8$

104. A variable line $ax + by + c = 0$, where a, b, c are in $A.P.$, is normal to a circle $(x-\alpha)^2 + (y-\beta)^2 = \gamma$, which is orthogonal to circle $x^2 + y^2 - 4x - 4y - 1 = 0$. The value of $\alpha + \beta + \gamma$ is equal to

(A) 3 (B) 5 (C) 10 (D) 7

Ans. : d

$ax + by + c = 0$
$a + c = 2b \Rightarrow a - 2b + c = 0$
$x = 1, y = -2$
$(1,-2) = (\alpha, \beta)$
$(x-1)^2 + (y+2)^2 = \gamma$
$\Rightarrow x^2 + y^2 - 2x + 4y + 5 - \gamma = 0$
it is orthogonal to $x^2 + y^2 - 4x - 4y - 1 = 0$
$\Rightarrow 4 - 8 = 5 - \gamma - 1$
$\gamma = 8$
$\alpha + \beta + \gamma = 1 - 2 + 8 = 7$

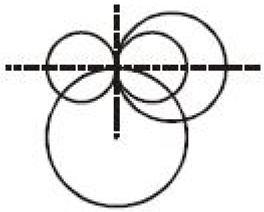
105. Let $S = 0$ is the locus of centre of a variable circle which intersect the circle $x^2 + y^2 - 4x - 6y = 0$ orthogonally at $(4,6)$. If P is a variable point of $S = 0$, then

least value of OP is (where O is origin)

(A) $\sqrt{13}$ (B) $2\sqrt{13}$ (C) 10 (D) 13

Ans. : b

$2x+3y=26$



106. Consider the equation of circles

$S_1: x^2+y^2+24x-10y+a=0$

$S_2: x^2+y^2=36$ which of the following is not correct

(A) Number of non-negative integral values of $'a'$ such that $S_1=0$ represents a real circle 170

(B) If $S_1=0$ and $S_2=0$ has no point in common, then number of integral values of a is more than 49

(C) If $S_1=0$ and $S_2=0$ intersect orthogonally then $a=36$

(D) If $a=0$, then number off common tangents to the circles $S_1=0$. and $S_2=0$ are 3

Ans. : d

(1) $S_1 \equiv x^2+y^2+24x-10y+a=0$

for real circle, $g^2+f^2-c \geq 0$

$144+25-a \geq 0$

$a \leq 169$

Also $a \geq 0$

$\therefore$ Total non-negative integral values of

$a=170$

(2) for no point in common $c_1c_2 > r_1+r_2$

& $c_1c_2 < |r_1-r_2|$

$c_1c_2=13$

$13 > \sqrt{169-a}+6$

$\Rightarrow \quad 169-a < 49$

$a > 120$ and $a \leq 169$

So in this condition we have 49 integral values of a

But from $c_1c_2 < |r_1-r_2|$,

we will get additional values of a. So

B is false

(3) for orthogonal cut

$2.12.0+2(-5).0=-36+a$

$\Rightarrow \quad a=36$

(4) If $a=0, c_1c_2=13$ and $r_1+r_2=19$

$c_1c_2<r_1+r_2$

No. of common tangent $=2$

107. The length of common chord of the circles $x^2+y^2+2x+4y-20=0$ and $x^2+y^2+6x-8y+10=0$ is

(A) $5\sqrt{\frac{3}{2}}$ (B) $2\sqrt{\frac{3}{2}}$ (C) 5 (D) $\frac{5\sqrt{5}}{2}$

Ans. : a

$r_1=5, \quad r_2=\sqrt{15}$

$\mathrm{C_1C_2}=\sqrt{4+36}=\sqrt{40}$

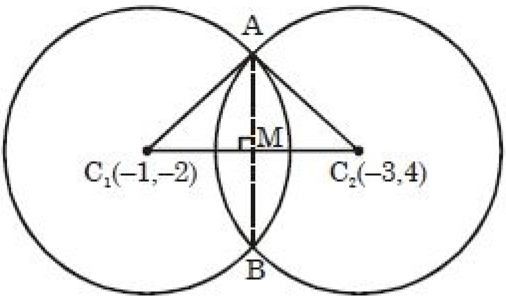
common chord $\mathrm{S_1-S_2}=0$

$\Rightarrow \quad 2x-6y+15=0$

$\mathrm{C_1M}=\mathrm{p}=\left|\frac{-2+12+15}{\sqrt{40}}\right|=\frac{25}{\sqrt{40}}$

$AB=2\sqrt{25-\frac{625}{40}}=2\sqrt{\frac{375}{40}}$

$=2\sqrt{\frac{75}{8}}=5\sqrt{\frac{3}{2}}$



108. If from origin two tangents are drawn to the circle $(x-2)^2+y^2=1$, then length of chord of contact is-

(A) 1 (B) $\frac{1}{2}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$

Ans. : c

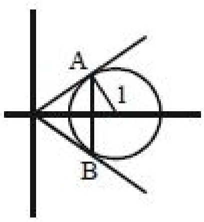
$x^2+y^2-4x+3=0$

equation of chord of contact AB is

$-2(x+0)+3=0$

$x=\frac{3}{2}$

length of chord $=2\sqrt{1-\frac{1}{4}}=\sqrt{3}$



109. Tangents drawn from origin O two the circle $x^2+y^2+2gx+2fy+c\ = 0$ touch the circle at the points P and Q. Then the equation of the circumcircle of the triangle OPQ is

(A) $x^2+y^2+2gx+2fy=0$ (B) $x^2+y^2+gx+fy=0$

(C) $x^2+y^2-gx-fy=0$ (D) $x^2+y^2-2gx-2fy=0$

Ans. : b

The equation of the chord of contact of tangents drawn from the origin to the circle $x^2+y^2+2gx+2fy+c=0$ is

$gx+fy+c=0$ ....(i)

The required circle passes through the intersection of the given circle and line (i).
Therefore, its equation is

$\left(x^2+y^2+2gx+2fy+c\right)+\lambda(gx+fy+c)=0$ ....(ii)

this passes through $(0,0)$

$\therefore c+\lambda c=0 \Rightarrow \lambda=-1$

putting $\lambda=-1$ in (ii), the eq. of the req. circle is

$x^2+y^2+gx+fy=0.$

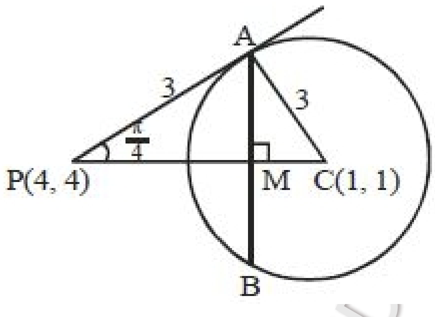
110. Length of chord of contact of tangents drawn from the point $(4,4)$ to the circle $x^2+y^2-2x-2y-7=0$ is-

(A) $2\sqrt{2}$ (B) $3\sqrt{2}$ (C) $4\sqrt{2}$ (D) $5\sqrt{2}$

Ans. : b

$AM=3\,\sin\frac{\pi}{4}=\frac{3}{\sqrt{2}}$

$AB=2AM=2\times\frac{3}{\sqrt{2}}=3\sqrt{2}$



111. The circumference of the circle $x^2+y^2-2x+8y-q=0$ is bisected by the circle $x^2+y^2+4x+12y+p=0$, then $p+q$ is equal to

(A) 25 (B) 100 (C) 10 (D) 48

Ans. : c

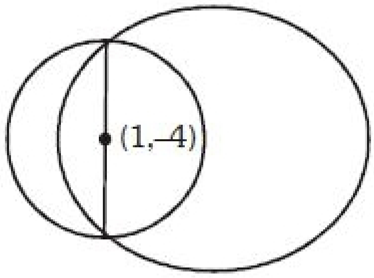
Common chord of given circle

$6x+4y+(p+q)=0$

which is diameter of $x^2+y^2-2x+8y-q=0$

centre $(1,-4)$

$6-16+(p+q)=0 \quad \Rightarrow p+q=10$



112. The common chord of two intersecting circles c_1 & c_2 can be seen from their centres at the angles of 90^o and 60^o respectively . If the distance between their centres is equal to $\sqrt{3}+1$ then the radii of c_1 & c_2 are :

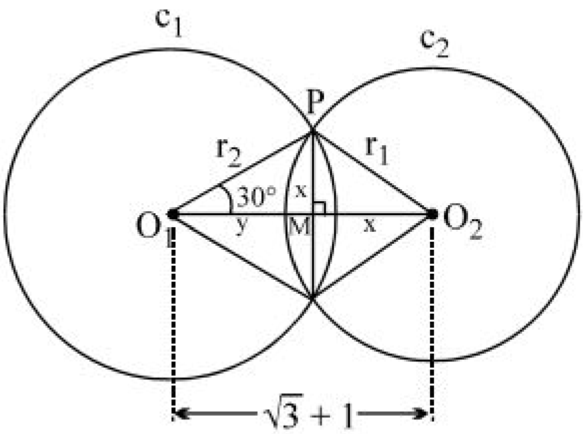
(A) $\sqrt{3}$ & 3 (B) $\sqrt{2}$ & $2\sqrt{2}$ (C) $\sqrt{2}$ & 2 (D) $2\sqrt{2}$ & 4

Ans. : c

$\frac{y}{\sin\ 60^\circ}=\frac{x}{\sin\ 30^\circ} \Rightarrow\ y=\sqrt{3}x$

and $x\left(\sqrt{3}\ +\ 1\right)=\sqrt{3}+1\ \Rightarrow\ x=1$

$\Rightarrow\ r_1=x\sqrt{2}=\sqrt{2}$ and $r_2=2$



113. Two circles whose radii are equal to 4 and 8 intersect at right angles. The length of their common chord is

(A) $\frac{16}{\sqrt{5}}$ (B) 8 (C) $4\sqrt{6}$ (D) $\frac{8\sqrt{5}}{5}$

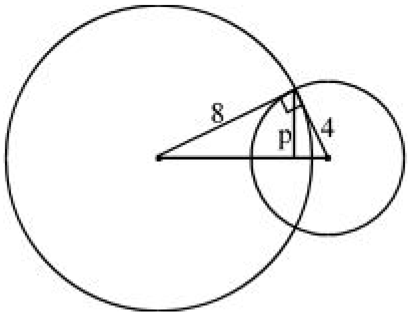
Ans. : a

distance $'d'$ between the centres is $=\sqrt{8^2\ +\ 4^2}\ =4\sqrt{5}$

$\Rightarrow\ l_e=\sqrt{d_2\ -\ (r_1\ -\ r_2)^2}=\sqrt{80\ -\ 16}=8.$

Also $4\sqrt{5}.p=8.4\ \Rightarrow\ p=\frac{8}{\sqrt{5}}$

$\Rightarrow$ length of common chord is $\frac{16}{\sqrt{5}}$



114. The chord of contact of the tangents drawn from a point on the circle, $x^2+y^2=a^2$ to the circle $x^2+y^2=b^2$ touches the circle $x^2+y^2=c^2$ then a,b,c are in :

(A) $A.P.$ (B) $G.P.$ (C) $H.P.$ (D) $A.G.P.$

Ans. : b

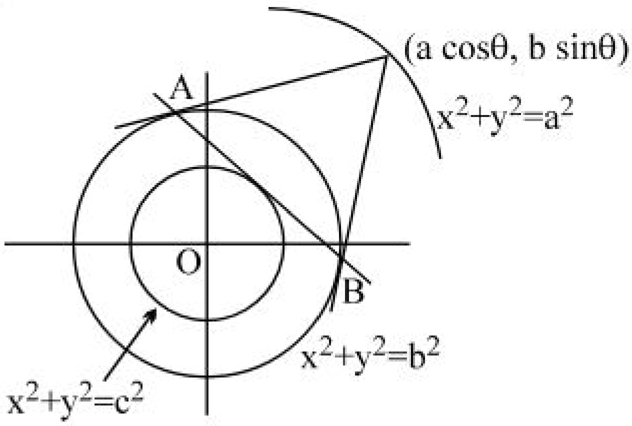
equation of chord of contact AB

$x\ a\ cos\theta\ +\ y\ a\ sin\theta\ = b^2.....(1)$

this is tangent to the circle $x^2+y^2=c^2$

$\therefore$ perpendicular from $(0,0)$ on (1) is equal to c

$\Rightarrow \frac{b^2}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}=c\ \ \Rightarrow b^2=ac$



115. The distance between the chords of contact of tangents to the circle ; $x^2+y^2+2gx+2fy+c=0$ from the origin & the point (g,f) is :

(A) $\sqrt{g^2\ +\ f^2}$ (B) $\frac{\sqrt{g^2\ +\ f^2\ -\ c}}{2}$ (C) $\frac{g^2\ +\ f^2\ -\ c}{2\sqrt{g^2\ +\ f^2}}$ (D) $\frac{\sqrt{g^2\ +\ f^2\ +\ c}}{2\sqrt{g^2\ +\ f^2}}$

Ans. : c

Equation of chord or contact are

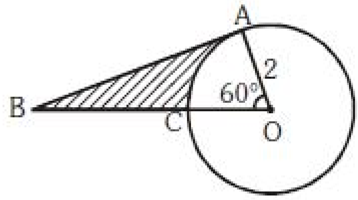
$gx+fy+c=0....(1)$

& $2gx+2fy+\frac{g^2+f^2+c}{2}=0....(2)$

These lines are parallel

hence distance = $\left|\frac{c-\frac{g^2+f^2+c}{2}}{\sqrt{g^2+f^2}}\right|$

116. In the given figure, AB is tangent to the circle with centre O , the ratio of the shaded region to the unshaded region of the triangle OAB is



(A) $\frac{2\sqrt{3}-2}{\pi}$ (B) $\frac{3\sqrt{3}-2}{\pi}$ (C) $\frac{2-\sqrt{3}}{\pi}$ (D) $\frac{3\sqrt{3}}{\pi}-1$

Ans. : d

In $\Delta\mathrm{AOB}, \mathrm{AB}=2\tan 60^{\circ}=2\sqrt{3}$

$\Rightarrow$ Area of $\Delta\mathrm{AOB}=\frac{1}{2}\times 2\times 2\sqrt{3}=2\sqrt{3}$

Area of sector $\mathrm{OAC}=\frac{60}{360}\pi(2)^2=\frac{2\pi}{3}$

$\Rightarrow$ Ratio $=\frac{2\sqrt{3}-\frac{2\pi}{3}}{\frac{2\pi}{3}}=\frac{3\sqrt{3}}{\pi}-1$

117. The angle between the pair of tangents from the point $(1,1/2)$ to the circle $x^2+y^2+4x+2y-4=0$ is-

(A) $\cos^{-1}\frac{4}{5}$ (B) $\sin^{-1}\frac{4}{5}$ (C) $\sin^{-1}\frac{3}{5}$ (D) None of these

Ans. : b

$\mathrm{r}=3$

$\sqrt{S_1}=\frac{3}{2}$

$2\propto=2\tan^{-1}\left(\frac{r}{\sqrt{S_1}}\right)$

$=2\tan^{-1}\left(\frac{3}{3/2}\right)=2\tan^{-1}(2)=\sin^{-1}\left(\frac{4}{5}\right)$

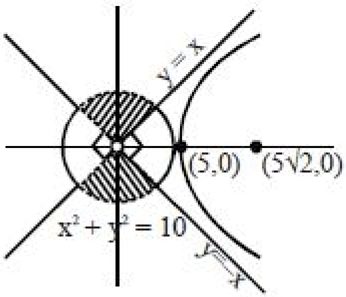
118. Number of integral points interior to the circle $x^2+y^2=10$ from which exactly one real tangent can be drawn to the curve $\sqrt{\left(x+5\sqrt{2}\right)^2+y^2}-\sqrt{\left(x-5\sqrt{2}\right)^2+y^2}=10$ are (where integral point (x,y) means $x,y\in I$)

(A) 12 (B) 14 (C) 16 (D) 18

Ans. : b

All the points must lie in the shaded figure where $|y|\geq|x|$ and equality holds if $x>0$

such that $x^2+y^2<10$



119. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2+y^2=4$ at $(1,\sqrt{3})$ is

(A) $2\sqrt{3}$ (B) $\sqrt{3}$ (C) $1/\sqrt{3}$ (D) 1

Ans. : (B) $\sqrt{3}$

120. Consider circle $S: x^2+y^2=1$ and $P(0,-1)$ on it. A ray of light gets reflected from tangent to S at P from the point with abscissa -3 and becomes tangent to the circle S. Equation of reflected ray is

(A) $3x+4y-5=0$ (B) $-3x+4y+5=0$ (C) $3x-4y+5=0$ (D) $3x-4y-5=0$

Ans. : c

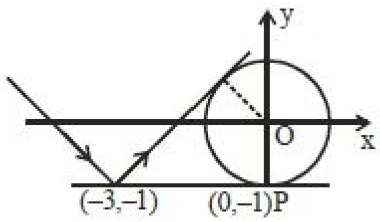
$y+1=m(x+3)$

$\Rightarrow mx-y+3x-1=0$

Distance from $(0,0)=1$

$\Rightarrow \left|\frac{3m-1}{\sqrt{1+m^2}}\right|=1 \Rightarrow m=0$ or $\frac{3}{4}$

$\therefore$ Reflected ray : $3x-4y+5=0$



121. The focus of the parabola $4y^2-6x-4y=5$ is

(A) $(-8/5,2)$ (B) $(-5/8,1/2)$ (C) $(1/2,5/8)$ (D) $(5/8,-1/2)$

Ans. : b

(b) Given equation of parabola written in standard form, we get

$4\left(y-\frac{1}{2}\right)^2=6(x+1)$

$\Rightarrow \left(y-\frac{1}{2}\right)^2=\frac{3}{2}(x+1)$

$\Rightarrow Y^2=\frac{3}{2}X$

where, $Y=y-\frac{1}{2},\ X=x+1$

$\therefore y=Y+\frac{1}{2},\ x=X-1$ .....(i)

For focus $X=a,\ Y=0$

$\because 4a=\frac{3}{2} \Rightarrow a=\frac{3}{8} \Rightarrow x=\frac{3}{8}-1=-\frac{5}{8}$
$y=0+\frac{1}{2}=\frac{1}{2}$, Focus$=\left(-\frac{5}{8},\frac{1}{2}\right)$.

122. The latus rectum of the parabola $y^2=5x+4y+1$ is

(A) $\frac{5}{4}$ (B) 10 (C) 5 (D) $\frac{5}{2}$

Ans. : c

(c) $y^2-4y+4=5x+5$

$(y-2)^2=5(x+1)$

Obviously, latus rectum is 5.

123. The equation of the latus rectum of the parabola represented by equation $y^2+2Ax+2By+C=0$ is

(A) $x=\frac{B^2+A^2-C}{2A}$ (B) $x=\frac{B^2-A^2+C}{2A}$ (C) $x=\frac{B^2-A^2-C}{2A}$ (D) $x=\frac{A^2-B^2-C}{2A}$

Ans. : c

(c) $(y+B)^2=-2Ax-C+B^2=-2A\left(x+\frac{C}{2A}-\frac{B^2}{2A}\right)$

Equation of latus rectum $x+\lambda=0$

Vertex $=\left(\frac{-C+B^2}{2A},B\right)$, focus $\equiv\left(\frac{-C+B^2}{2A}-\frac{A}{2},B\right)$

Equation of $L.R.$ is $x=\frac{-C+B^2}{2A}-\frac{A}{2}=\frac{B^2-A^2-C}{2A}$.

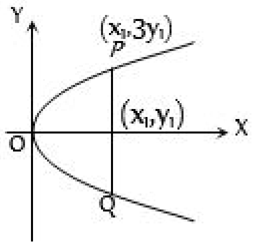
124. PQ is a double ordinate of the parabola $y^2=4ax$. The locus of the points of trisection of PQ is

(A) $9y^2=4ax$ (B) $9x^2=4ay$ (C) $9y^2+4ax=0$ (D) $9x^2+4ay=0$

Ans. : a

(a) Required locus is $(3y)^2=4ax$

==> $9y^2=4ax$.

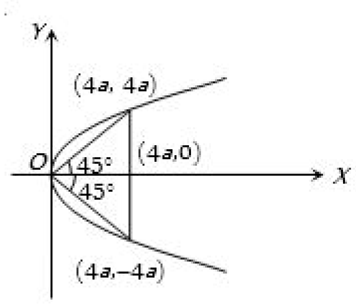


125. If a double ordinate of the parabola $y^2=4ax$ be of length $8a$, then the angle between the lines joining the vertex of the parabola to the ends of this double ordinate is ....$^\circ$

(A) 30 (B) 60 (C) 90 (D) 120

Ans. : c

(c) It is clear from figure.



126. The locus of the intersection point of $x\cos\alpha - y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ is

(A) Ellipse (B) Hyperbola (C) Parabola (D) None of these

Ans. : d

(d) $x\ \cos\alpha - y\sin\alpha = a,\ x\ \sin\alpha - y\cos\alpha = b$ Intersection points are

$h = \frac{a\ \cos\alpha - b\ \sin\alpha}{\cos 2\alpha}$, $k = \frac{a\ \sin\alpha - b\cos\alpha}{\cos 2\alpha}$

Then the locus of point (h, k) is $x^4 + y^4 - 2x^2y^2\ = (a^2 + b^2)\ (x^2 + y^2) + 4abxy$, which is not a locus of any given curves.

127. The equation of the hyperbola whose directrix is $2x + y = 1$, focus $(1,1)$ and eccentricity $= \sqrt{3}$, is

(A) $7x^2 + 12xy - 2y^2 - 2x + 4y - 7 = 0$

(B) $11x^2 + 12xy + 2y^2 - 10x - 4y + 1 = 0$

(C) $11x^2 + 12xy + 2y^2 - 14x - 14y + 1 = 0$

(D) None of these

Ans. : a

(a) $S\,(1, 1)$, directrix is $2x + y = 1$ and $e = \sqrt{3}$.

Now let the various point be (h,k),

then accordingly $\frac{\sqrt{(h-1)^2 + (k-1)^2}}{\frac{2h+k-1}{\sqrt{5}}} = \sqrt{3}$

Squaring both the sides, we get

$5[(h-1)^2 + (k-1)^2] = 3(2h + k - 1)^2$

On simplification, the required locus is

$7x^2 + 12xy - 2y^2 - 2x + 4y - 7 = 0$.

128. The equation of the directrices of the conic $x^2 + 2x - y^2 + 5 = 0$ are

(A) $x = \pm 1$ (B) $y = \pm 2$ (C) $y = \pm\sqrt{2}$ (D) $x = \pm\sqrt{3}$

Ans. : c

(c) $(x + 1)^2 - y^2 - 1 + 5 = 0$

$\Rightarrow\ -\frac{(x+1)^2}{4} + \frac{y^2}{4} = 1$

Equation of directrices of $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ are $y = \pm\frac{b}{e}$

Here $b = 2,\ e = \sqrt{1+1} = \sqrt{2}$

Hence $y=\pm\frac{2}{\sqrt{2}}$

$\Rightarrow y=\pm\sqrt{2}$.

129. The equation $13[(x-1)^2+(y-2)^2]=3(2x+3y-2)^2$ represents

(A) Parabola (B) Ellipse (C) Hyperbola (D) None of these

Ans. : c

(c) Here coefficient of x^2 is $+ve$ and that of y^2 is $-ve$

$i.e.$, a hyperbola.

130. A point ratio of whose distance from a fixed point and line $x=9/2$ is always $2:3$. Then locus of the point will be

(A) Hyperbola (B) Ellipse (C) Parabola (D) Circle

Ans. : b

(b) In question, $PS=\frac{2}{3}PM$ (Given)

Focus $S(-2,0)$,

Equation of directrix $2x-9=0$

$(PS)^2=\frac{4}{9}(PM)^2$

==> $(h+2)^2+(k)^2=\frac{4}{9}\left(\frac{2h-9}{2}\right)^2$

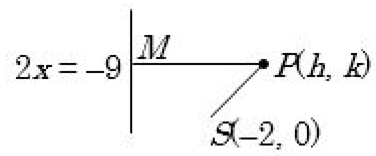
==> $9[(h+2)^2+(k)^2]=\frac{4(2h-9)^2}{4}$

==> $9h^2+9k^2+36h+36=4h^2+81+36h$

==> $\frac{5h^2}{45}+\frac{9k^2}{45}=1$

==> $\frac{h^2}{9}+\frac{k^2}{5}=1$

Locus of point $P(h,k)$ is $\frac{x^2}{9}+\frac{y^2}{5}=1$, which is an ellipse



131. The eccentricity of the conic $4x^2+16y^2-24x-3y=1$ is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{4}$ (D) $\sqrt{3}$

Ans. : a

(a) Given equation of conic is $4x^2+16y^2-24x-3y=1$

$\Rightarrow (2x-6)^2+(4y-4)^2=53$

$\Rightarrow 4\,(x-3)^2+16\,(y-1)^2=53$

$\Rightarrow \frac{(x-3)^2}{53/4}+\frac{(y-1)^2}{53/16}=1$

$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{53/16}{53/4}}$

$= \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$.

132. The equation $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ represents

(A) A circle
(B) An ellipse
(C) A hyperbola
(D) A rectangular hyperbola

Ans. : b

(b) Check $\Delta \neq 0$ and $h^2 < ab$.

133. The points of intersection of the curves whose parametric equations are $x = t^2 + 1,\ y = 2t$ and $x = 2s,\ y = \frac{2}{s}$ is given by

(A) $(1,\ -3)$
(B) $(2, 2)$
(C) $(-2, 4)$
(D) $(1, 2)$

Ans. : b

(b) Eliminating t from $x = t^2 + 1,\ y = 2t,$ we obtain $y^2 = 4x - 4$

Similarly eliminating s from $x = 2s, y = \frac{2}{s}$,

we get $xy = 4$.

Hence point of intersection is $(2, 2)$.

134. Curve $16x^2 + 8xy + y^2 - 74x - 78y + 212 = 0$ represents

(A) Parabola
(B) Hyperbola
(C) Ellipse
(D) None of these

Ans. : a

(a) $\Delta \neq 0, h^2 = ab\ \ i.e.$, parabola.

135. The equation $x^2 - 2xy + y^2 + 3x + 2 = 0$ represents

(A) A parabola
(B) An ellipse
(C) A hyperbola
(D) A circle

Ans. : a

(a) $\Delta = (1)(1)(2) + 2\left(\frac{3}{2}\right)(0)(-1) - (1)(0)^2 - (1)\left(\frac{3}{2}\right)^2 - 2(-1)^2$

$= 2 - \frac{9}{4} - 2 < 0$ and $h^2 - ab = 1 - 1 = 0$.

i.e., $h^2 = ab$ ==> a parabola.

136. The equation $y^2 - x^2 + 2x - 1 = 0$ represents

(A) A hyperbola
(B) An ellipse
(C) A pair of straight lines
(D) A rectangular hyperbola

Ans. : c

(c) Given equation is $y^2 - x^2 + 2x - 1 = 0$

Comparing the given equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

We get, $a = 1$, $h = 0$, $b = 1$, $g = 1$, $f = 0$, $c = -1$

$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$\Delta = 1 + 0 + 0 - 1 = 0$

Hence, the given equation represents two straight lines.

137. Equation $\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=4$ represents

(A) Parabola
(B) Ellipse
(C) Circle
(D) Pair of straight lines

Ans. : d

(d) Given equation is , $\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=4$

$\sqrt{(x-2)^2+y^2}=4-\sqrt{(x-2)^2+y^2}$

Squaring both sides, we get $\sqrt{(x+2)^2+y^2}=x+2$

Again squaring both sides, we get $y^2=0$, which is the equation of pair of straight lines.

138. The centre of $14x^2-4xy+11y^2-44x-58y+71=0$

(A) $(2,3)$
(B) $(2,-3)$
(C) $(-2,3)$
(D) $(-2,-3)$

Ans. : a

(a) Use formula of centre of conic

$i.e.,\left(\frac{hf-bg}{ab-h^2},\frac{gh-af}{ab-h^2}\right)$.

139. For all real values of m, the straight line $y=mx+\sqrt{9m^2-4}$ is a tangent to the curve :

(A) $9x^2+4y^2=36$
(B) $4x^2+9y^2=36$
(C) $9x^2-4y^2=36$
(D) $4x^2-9y^2=36$

Ans. : d

$(mx-y)^2=9m^2-4$

$m^2x^2-2mxy+y^2=9m^2-4$

$m^2(x^2-9)-2mxy+y^2+4=0$

$D=0\Rightarrow 4x^2y^2=4(x^2-9)(y^2+4)$

$x^2y^2=x^2y^2+4x^2-9y^2-36$

$4x^2-9y^2=36\Rightarrow D$

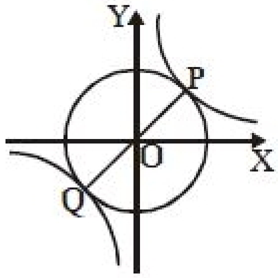
140. The curve $xy=c,(c>0)$, and the circle $x^2+y^2=1$ touch at two points. Then the distance between the points of contacts is

(A) 1
(B) 2
(C) $2\sqrt{2}$
(D) None of these

Ans. : b

Form the diagram,

$PQ=$ Diameter of circle $=2$



141. The equation of a line passing through the centre of a rectangular hyperbola is $x-y-1=0$. If one of the asymptotes is $3x-4y-6=0$, the equation of other asymptote is

(A) $4x-3y+17=0$
(B) $-4x-3y+17=0$
(C) $-4x+3y+1=0$
(D) $4x+3y+17=0$

Ans. : d

We know that asymptotes of rectangular hyperbola is mutually perpendicular, thus other asymptote should be $4\mathrm{x}+3\mathrm{y}+\lambda=0$

Also, intersection point of asymptotes is also the centre of the hyperbola.

Hence, intersection point of $4\mathrm{x}+3\mathrm{y}+\lambda=0$ and

$3x-4y-6=0$ is $\left(\frac{18-4\lambda}{25},\frac{-12\lambda-96}{100}\right)$ and it

should lie on the line $x-y-1=0$

$\therefore \frac{18-4\lambda}{25}-\frac{-12\lambda-96}{100}-1=0$

$\Rightarrow \lambda=17$

Hence, equation of other asymptote is

$4x+3y+17=0$

142. The number of possible tangents which can be drawn to the curve $4x^2-9y^2=36$, which are perpendicular to the straight line $5x+2y-10=0$ is

(A) 0
(B) 1
(C) 2
(D) 4

Ans. : a

Slope of tangent $=\frac{2}{5}$

Now $c^2=a^2m^2-b^2$

$=9\left(\frac{2}{5}\right)^2-4<0$

No such tangent exist

143. Let P is a point on hyperbola $x^2-y^2=4$, which is at minimum distance from $(0,-1)$ then distance of P from $x-$ axis is

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $\sqrt{2}$

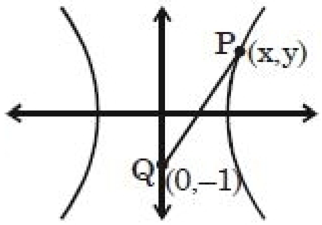
Ans. : b

$\mathrm{PQ}=\sqrt{\mathrm{x}^2+(\mathrm{y}+1)^2}$

$=\sqrt{y^2+4+y^2+2y+1}$

$= \sqrt{2\left(y+\frac{1}{2}\right)^2+\frac{9}{2}}$

PQ is minimum if $y=-\frac{1}{2}$



144. If for a hyperbola the ratio of length of conjugate Axis to the length of transverse axis is $3:2$ then the ratio of distance between the focii to the distance between the two directrices is

(A) $13:4$ (B) $4:13$ (C) $\sqrt{13}:2$ (D) $2:\sqrt{13}$

Ans. : a

$\frac{b}{a}=\frac{3}{2}$,

Now $\frac{2ae}{\frac{2a}{e}}=e^2=1+\frac{b^2}{a^2}$

$=1+\frac{9}{4}=\frac{13}{4}$

145. The locus of middle points of the chords of the circle $x^2+y^2=a^2$ which touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(A) $(x^2-y^2)^2=a^2x^2+b^2y^2$ (B) $(x^2+y^2)^2=a^2x^2+b^2y^2$

(C) $(x^2-y^2)^2=a^2x^2-b^2y^2$ (D) $(x^2+y^2)^2=a^2x^2-b^2y^2$

Ans. : d

Equation of chord of the given circle whose middle point is

(h,k) is $xh+yk=h^2+k^2$

and equation of tangent line

upon hyperbola is $y-mx=\sqrt{a^2m^2-b^2}$

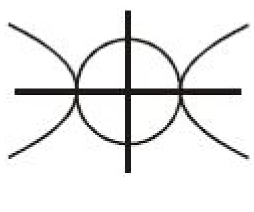
As both lines are identical

$\Rightarrow -\frac{m}{h}=\frac{1}{k}=\frac{\sqrt{a^2m^2-b^2}}{h^2+k^2}$

Eliminating m

$\Rightarrow \left(h^2+k^2\right)^2=\left(a^2h^2-b^2k^2\right)$

hence the locus is $\left(x^2+y^2\right)^2=\left(a^2x^2-b^2y^2\right)$



146. $P(6,3)$ is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at point P intersect the $x-$ axis at $(10,0)$, then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{3}}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\sqrt{\frac{5}{2}}$ (D) $\frac{\sqrt{13}}{2}$

Ans. : a

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Slope of normal is $-\frac{a^2}{2b^2}$

$\therefore$ Equation of normal at $\mathrm{P}(6,3)$ is

$y-3=-\frac{a^2}{2b^2}(x-6)$

at $(10,0) \Rightarrow -3=-\frac{a^2}{2b^2}(10-6)$

$\Rightarrow \quad \frac{3}{2}=\frac{a^2}{b^2}$

$\therefore \quad e^2=1+\frac{b^2}{a^2}=1+\frac{2}{3}=\frac{5}{3}$

$\therefore \quad e=\sqrt{\frac{5}{3}}$

147. The graph of the conic $x^2-(y-1)^2=1$ has one tangent line with positive slope that passes through the origin. The point of the tangency being (a,b) then find the value of $\sin^{-1}\left(\frac{a}{b}\right)$

(A) $\frac{5\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Ans. : d

differentiate the curve

$2x-2(y-1)\frac{dy}{dx}=0$

$\left.\frac{dy}{dx}\right]_{a,\,b}=\frac{a}{b-1}=(m_{OP}=)$

$a^2=b^2-b\ldots(1)$

Also (a,b) satisfy the curve

$a^2-(b-1)^2=1$

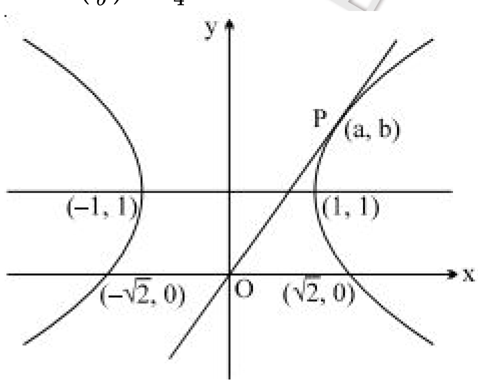
$a^2-(b^2-2b+1)=1$

$a^2-b^2+2b=2$

$-b+2b=2 \Rightarrow b=2$

$a=\sqrt{2} \quad (a\neq-\sqrt{2})$

$\sin^{-1}\left(\frac{a}{b}\right)=\frac{\pi}{4}$



148. Area of the quadrilateral formed with the foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(A) $4(a^2+b^2)$ (B) $2(a^2+b^2)$ (C) (a^2+b^2) (D) $\frac{1}{2}(a^2+b^2)$

Ans. : b

Given hyperbolas are conjugate and the quadrilateral formed by their foci is a square

now $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

$e_1^2=1+\frac{b^2}{a^2}$; $e_2^2=1+\frac{a^2}{b^2}$;

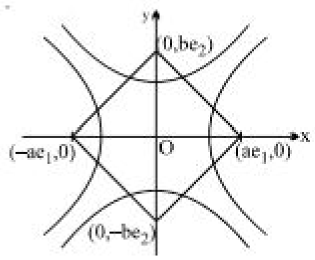
$e_1^2\,e_2^2=\frac{(a^2+b^2)^2}{a^2b^2}$;

$e_1e_2=\frac{a^2+b^2}{ab}$

$A=\frac{(2ae_1)(2be_2)}{2}$

$=2abe_1e_2$

$=\frac{2ab(a^2+b^2)}{ab}$



149. If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ of eccentricity $e=\sqrt{3}$ from its asymptotes is equal to 6, then the length of the transverse axis of the hyperbola is

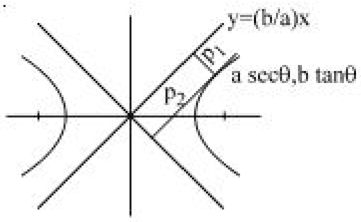
(A) 3 (B) 6 (C) 8 (D) 12

Ans. : b

$p_1p_2=\frac{a^2b^2}{a^2+b^2}=\frac{a^2.\,a^2\,(e^2-1)}{a^2\,e^2}=6$

$\frac{2a^2}{3}=6\Rightarrow a^2=9\Rightarrow a=3$

hence $2a=6$



150. With one focus of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ as the centre , a circle is drawn which is tangent to the hyperbola with no part of the circle being outside the hyperbola. The radius of the circle is

(A) $less\ than\ 2$ (B) 2 (C) $\frac{11}{3}$ (D) $none$

Ans. : b

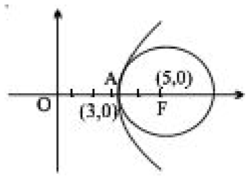
$e^2 = 1 + \frac{16}{9} \;=\; \frac{25}{9}$

$\Rightarrow e = \frac{5}{3}$

$\therefore$ focus $= (5, 0)$

Use reflection property to prove that circle cannot touch at two points.

It can only be tangent at the vertex $r = 5 - 3 = 2$



*** Given section consists of questions of 2 marks each.** **[34]**

151. Find the equation of the circle with centre (0, 2) and radius 2

Ans. : Here h = 0, k = 2 and r = 2

The equation of circle is

$(x - h)^2 + (y - k)^2 = r^2$

$\therefore (x - 0)^2 + (y - 2)^2 = (2)^2$

$\Rightarrow x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

Which is required equation of circle.

152. Find the equation of the circle with centre (-a, -b) and radius $\sqrt{a^2 - b^2}$

Ans. : Here h = -a, k = -b and r = $\sqrt{a^2 - b^2}$

The equation of circle is

$(x - h)^2 + (y - k)^2 = r^2$

$\therefore (x + a)^2 + (y + b)^2 = \left(\sqrt{a^2 - b^2}\right)^2$

$\Rightarrow x^2 + a^2 + 2ax + y^2 + b^2 + 2by = a^2 - b^2$

$\Rightarrow x^2 + y^2 + 2ax + 2by + 2b^2 = 0$

Which is required equation of circle.

153. Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Ans. : The equation of circle is

$(x - h)^2 + (y - k)^2 = r^2 \ . . . \text{(i)}$

Since the circle passes through point (4, 5) and co-ordinates of centre are (2, 2).

$\therefore$ radius of circle $= \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$

Now the equation of required circle is

(x - 2)^2 + (y - 2)^2 = $(\sqrt{13})^2$ $\Rightarrow$ x^2 + 4 - 4x + y^2 + 4 - 4y = 13

$\Rightarrow$ x^2 + y^2 - 4x - 4y - 5 = 0

154. Find the coordinates of the foci, and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

Ans. : The equation of given hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ which is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The foci and vertices of the hyperbola lie on x-axis.

$\therefore$ a^2 = 16 $\Rightarrow$ a = 4 and b^2 = 9 $\Rightarrow$ b = 3

Now c^2 = a^2 + b^2 = 16 + 9 = 25 ⇒ c = 5

$\therefore$ Coordinates of foci are (± c, 0) i.e. (± 5, 0)

Coordinates of vertices are (± a, 0) i.e. (± 4, 0)

Eccentricity $(e) = \frac{c}{a} = \frac{5}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

155. Find the coordinates of the foci, and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

$\frac{y^2}{9} - \frac{x^2}{27} = 1$

Ans. : The equation of given hyperbola is $\frac{y^2}{9} - \frac{x^2}{27} = 1$ which is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

The foci and vertices of the hyperbola lie on x-axis.

a^2 = 9 ⇒ a = 3 and b^2 = 27 ⇒ b = 3$\sqrt{3}$

Now c^2 = a^2 + b^2 = 9 + 27 = 36 ⇒ c = 6

$\therefore$ Coordinates of foci are (0, ± c) i.e. (0, ± 6)

Coordinates of vertices are (0, ± a) i.e. (0, ± 3)

Eccentricity (e) = $= \frac{c}{a} = \frac{6}{3} = 2$

Length of latus rectum $= \frac{ab^2}{a} = \frac{2\times 27}{3} = 18$

156. Find the coordinates of the foci, and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas. 49y^2 - 16x^2 = 784

Ans. : The given equation of hyperbola is 49y^2 - 16x^2 = 784

i.e. $\frac{49y^2}{784} - \frac{16x^2}{784} = 1 \Rightarrow \frac{y^2}{16} - \frac{x^2}{49} = 1$ which is of the form $\frac{y^2}{a^2} - \frac{x^2}{a^2} = 1$

The foci and vertices of the hyperbola lie on y-axis.

$\therefore$ a^2 = 16 $\Rightarrow$ a = 4 and b^2 = 49 $\Rightarrow$ b = 7

Now c^2 = a^2 + b^2 = 16 + 49 = 65 $\Rightarrow$ c = $\sqrt{65}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ i.e. $(0, \pm\sqrt{65})$

Coordinates of vertices are $(0, \pm a)$ i.e $(0, \pm 4)$

Eccentricity $(e) = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$

157. Find the equation of the hyperbola, whose vertices (0, $\pm$3) and foci (0, $\pm$5).

Ans. : We have,

vertices = (0, $\pm$3) = (0, $\pm$a)

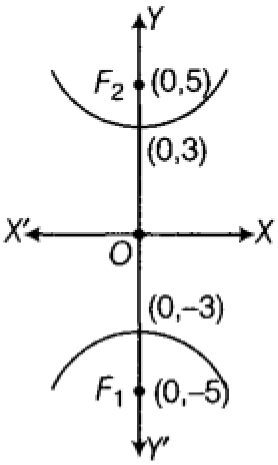
$\Rightarrow$ a = 3 and foci = (0, $\pm$c) = (0, $\pm$5)

$\Rightarrow$ c = 5

Also, we know that, $c^2 = a^2 + b^2$

$\Rightarrow 25 = 9 + b^2$ [$\because$ a = 3]

$\Rightarrow b^2 = 25 - 9 = 16$



Here, the foci and vertices lie on Y-axis,

Therefore equation of hyperbola is of the form

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

i.e., $\frac{y^2}{9} - \frac{x^2}{16} = 1$

158. Find the equation of hyperbola which has Vertices $(\pm 7, 0), e = \frac{4}{3}$

Ans. : Here vertices are (± 7, 0) which lie on x-axis.

So the equation of hyperbola in standard form is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ Vertices (± a, 0) is (± 7, 0) ⇒ a = 7

Now $e = \frac{4}{3} \Rightarrow \frac{c}{a} = \frac{4}{3} \Rightarrow \frac{c}{7} = \frac{4}{3} \Rightarrow c = \frac{28}{3}$

We know that $c^2 = a^2 + b^2$

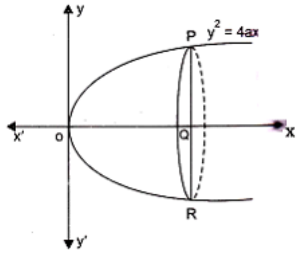
$\therefore \left(\frac{28}{3}\right)^2 = (7)^2 + b^2 \Rightarrow b^2 = \frac{784}{9} - 49 = \frac{343}{9}$

Thus required equation of hyperbola is

$\frac{x^2}{(7)^2} - \frac{y^2}{\frac{343}{9}} = 1 \Rightarrow \frac{x^2}{49} - \frac{9y^2}{343} = 1$

159. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

Ans. : A parabolic reflector with diameter PR = 20 cm and OQ = 5 cm.
Vertex of the parabola is (0, 0)
Let focus of the parabola be (a, 0).



Now PR = 20 cm ⇒ PQ = 10 cm
∴ Coordinate of point P are (5, 10)

Since the point lies on the parabola $y^2 = 4ax$

$\therefore (10)^2 = 4a \times 5 \Rightarrow a = \frac{100}{20} \Rightarrow$ a = 5
Thus required focus of the parabola is (5, 0).

160. Find the equation of the parabola which is symmetric about the y-axis, and passes through the point (2, -3).

Ans. : Since the parabola is symmetric about the y-axis and has its vertex at the origin, the equation is of the form $x^2 = 4ay$ or $x^2 = -4ay$,
But the parabola passes through (2,–3) which lies in the fourth quadrant, it must open downwards.

Thus the equation is of the form $x^2 = -4ay$
Since the parabola passes through (2, -3), we have
$2^2 = -4a(-3)$, i.e., $a = \frac{1}{3}$
Therefore, the equation of the parabola is

$x^2 = -4\left(\frac{1}{3}\right)y$, i.e., $3x^2 = -4y$

161. Find the coordinates of the centre and radius of each of the following circles:
$\mathrm{x}^2 + \mathrm{y}^2 - \mathrm{ax} - \mathrm{by} = 0$

Ans. : The given equation can be rewritten as $\mathrm{x}^2 + \mathrm{y}^2 = \frac{2\mathrm{ax}}{2} - \frac{2\mathrm{by}}{2} = 0$.

∴ Centre $= \left(\frac{\mathrm{a}}{2}, \frac{\mathrm{b}}{2}\right)$

And, radius $\sqrt{\left(\frac{\mathrm{a}}{2}\right)^2 + \left(\frac{\mathrm{b}}{2}\right)^2} = \frac{1}{2}\sqrt{\mathrm{a}^2 + \mathrm{b}^2}$

162. Find the equation of the circle whose centre is (1, 2) and which passes through the point (4, 6).

Ans. : We know that the equation of circle whose centre in (a, b) and radius r is
$(\mathrm{x} - \mathrm{a})^2 + (\mathrm{y} - \mathrm{b})^2 = \mathrm{r}^2 \ldots\ldots(1)$
We have centre = (1, 2)
$\therefore (\mathrm{x} - 1)^2 + (\mathrm{y} - 2)^2 \ldots\ldots(2)$

Also, circle passes through (4, 6)

$\therefore (4-1)^2 = (6-2)^2 = r^2$

$\Rightarrow 9 + 16 = r^2$

$\Rightarrow r = 5$

Thus, equation of required circle in

$(x-1)^2 + (y-2)^2 = 5^2$

or,

$x^2 + y^2 - 2x - 4y - 20 = 0$

163. Find the coordinates of the centre and radius of each of the following circles:

$\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0$

Ans. : The given equation can be rewritten as $x^2 + y^2 + 2x\cos\theta + 2y\sin\theta - 8 = 0.$

$\therefore$ Centre $= (-\cos\theta,\ \sin\theta)$

And, radius $= \sqrt{(-\cos\theta)^2(-\sin\theta)^2 + 8} = \sqrt{1+8} = 3$

164. Find the centre and radius of the following circles:

$x^2 + y^2 - 4x + 6y = 5$

Ans. : The general equation of orde is $(x-a)^2 + (y-b)^2 - r^2$

or $x^2 + y^2 - 2ax - 2by + a^2 + b^2 = r^2 \ldots\ldots(A)$

Where (a, b),s the centre and r be the radius of the circle.

$x^2 + y^2 - 4x + 6y = 5$

$x^2 + y^2 - 4x + 6y - 5$

$\Rightarrow (x^2 - 4x + 4) + (y^2 + 6y + 9) = 5 + 4 + 9$

$\Rightarrow (x-2)^2 + (y+3)^2 - (3\sqrt{2})^2$

Comparing with (A), we get

centre = (2, -3)

Radius $-3\sqrt{2}$

165. Find the equation of the circle passing through the point of intersection of the lines x + 3y = 0 and 2x - 7y = 0 and whose centre is the point of intersection of the lines x + y + 1 = 0 and x - 2y + 4 = 0.

Ans. : The given equations of lines are

x + 3y = 0 ....(1)

2x - 7 y = 0 ....(2)

x + y = -1 ....(3)

x - 2y = -4 .... (4)

The general equation of circle with centre (a, b) and radius r is

$(x - a)^2 + (y - b)^2 = r^2$ ....(A)

Centre of (A) is the point of intersection of (iii) & (iv)

$\therefore$ Centre = (-2, 1)

$\therefore$ (A)

⇒ $(x + 2)^2 + (y - 1)^2 = r^2$ .... (B)

Also, (A) passes through point of intersection of (1) & (2), that is through P = (0, 0)

$\therefore 2^2(-1)^2 = r^2 \Rightarrow r = \sqrt{5}$

Thus, the equation of required circle is

$(x + 2)^2 + (y - 1)^2 = 5$

or,

$x^2 + y^2 + 4X - 2y = 0$

166. If the line $y = \sqrt{3}x + +k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.
[**Hint:** Equate perpendicular distance from the centre of the circle to its radius]

Ans. : Given circle is $x^2 + y^2 = 16$

Center = (0, 0)

Radius r = 4

Perpendicular from the origin to the given line $y = \sqrt{3}x + k$ is equal to the radius.

$\therefore\ 4 = \left|\frac{0-0-k}{\sqrt{(1)^2+(\sqrt{3})^2}}\right| = \left|\frac{-k}{\sqrt{4}}\right|$

$\Rightarrow 4 = \pm\frac{k}{2}$

$\Rightarrow k = \pm 8$

Hence, the required values of k are ± 8

167. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

Ans. : Consider the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It is given that, length of latus rectum = half of minor axis

$\Rightarrow \frac{2b^2}{a} = b$

$\Rightarrow a = 2b$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 4b^2(1 - e^2)$

$\Rightarrow 1 - e^2 = \frac{1}{4}$

$\Rightarrow e^2 = \frac{3}{4}$

$\therefore\ e = \frac{\sqrt{3}}{2}$

*** Given section consists of questions of 3 marks each.** **[57]**

168. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$36x^2 + 4y^2 = 144$

Ans. : The equation of given ellipse is $36x^2 + 4y^2 = 144$

i.e. $\frac{36x^2}{144} + \frac{4y^2}{144} = 1 \Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1$

Now 36 > 4 ⇒ a^2 = 36 and b^2 = 4

So the equation of ellipse in standard form is $\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$

∴ a^2 = 36 ⇒ a = 6 and b^2 = 4 ⇒ b = 2

We know that $c=\sqrt{a^2-b^2}$

$\therefore c=\sqrt{36-4}=\sqrt{32}=4\sqrt{2}$

∴ Coordinates of foci are $(0,\ \pm c)$ i.e. $(0,\ \pm 4\sqrt{2})$

Coordinates of vertices are $(0,\ \pm a)$ i.e. $(0,\ \pm 6)$

Length of major axis = 2 a = $2\times 6=12$

Length of minor axis $=2b=2\times 2=4$

Eccentricity (e) $=\frac{c}{a}=\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$

Length of latus rectum $=\frac{ab^2}{a}=\frac{2\times 4}{6}=\frac{4}{3}$

169. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$4x^2 + 9y^2 = 36$

Ans. : The equation of given ellipse is $4x^2 + 9y^2 = 36$

i.e. $\frac{4x^2}{36}+\frac{9y^2}{36}=1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$

Now 9 > 4 ⇒ a^2 = 9 and b^2 = 4

So the equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

∴ a^2 = 9 ⇒ a = 3 and b^2 = 4 ⇒ b = 2

We know that $c=\sqrt{a^2-b^2}$

$c=\sqrt{9-4}=\sqrt{5}$

∴ Coordinates of foci are $(\pm c,\ 0)$ i.e. $(\pm\sqrt{5},\ 0)$

Coordinates of vertices are $(\pm a,\ 0)$ i.e. $(\pm 3,\ 0)$

Length of major axis $=2a=2\times 3=6$

Length of minor axis $2b=2\times 2=4$

Eccentricity (e) $=\frac{c}{a}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{3}=\frac{8}{3}$

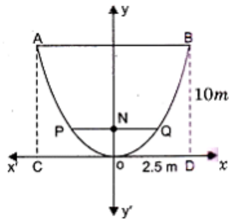
170. An arc is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

Ans. : Let AB be the parabolic arch having O as the vertex and OY as the axis.

The parabola is of the form x^2 = 4ay

Now CD = 5 m ⇒ OD = 2.5 m

BD = 10 m



$\Rightarrow$ Coordinates of point B are (2.5, 10)

Since the point B lies on the parabola $x^2 = 4ay$

$\therefore (2.5)^2 = 4a \times 10 \Rightarrow a = \frac{6.25}{40} = \frac{625}{4000} = \frac{5}{32}$

$\therefore$ Equation of parabola is $x^2 = 4 \times \frac{5}{32} y$

$\Rightarrow x^2 = \frac{5}{8} y$

Let PQ = d ⇒ NQ = $\frac{d}{2}$

$\therefore$ Coordinates of Point Q are $\left(\frac{d}{2}, 2\right)$

Since point Q lies on the parabola $x^2 = \frac{5}{8} y$

$\therefore \left(\frac{d}{2}\right)^2 = \frac{5}{8} \times 2 \Rightarrow \frac{d^2}{4} = \frac{5}{4} \Rightarrow d^2 = 5 \Rightarrow d = \sqrt{5}$

Thus width of arc $= \sqrt{5}\, m$ = 2.24m approx.

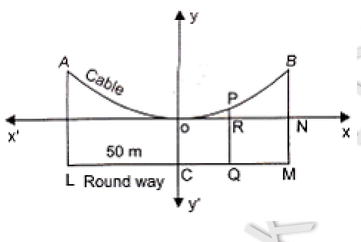
171. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Ans. : Let AOB be the cable of uniformly loaded suspension bridge. Let AL and BM be the longest wires of length 30 m each. Let OC be the shortest wire of length 6 m and LM be the roadway.

Now AL = BM = 30 m, OC = 6 m and LM = 100 m.

$\therefore$ LC = CM = $\frac{1}{2}$LM= 50 m

Let O be the vertex and axis of the parabola be y-axis. So the equation of parabola in standard form is $x^2 = 4ay$



Coordinates of point B are (50, 24)

Since point B lies on the parabola $x^2 = 4ay$

$\therefore (50)^2 = 4a \times 24 \Rightarrow a = \frac{2500}{4 \times 24} = \frac{625}{24}$

So equation of parabola is $x^2 = \frac{4 \times 625}{24} y \Rightarrow x^2 = \frac{625}{6} y$

Let length of the supporting wire PW at a distance of 18 m be h.

$\therefore$ OR = 18 m and PR = PQ – QP = PQ - OC = h - 6

Coordinates of point P are (18, h - 6)

Since the point P lies on parabola $x^2 = \frac{625}{6}y$

$\therefore$ $(18)^2 = \frac{625}{6}$ (h - 6) ⇒ 324 $\times$ 6 = 625h - 3750

⇒ 625 h = 1944 + 3750 ⇒ $h = \frac{5694}{625}$ = 9.11 m approx.

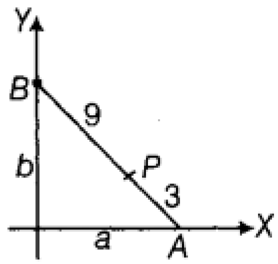
172. A rod of length 12 m moves with its ends always touching the coordinates axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the X-axis.

Ans. :

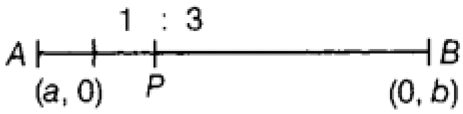
Let l be the length of the rod and which at any position meet X-axis at A (a, 0) and also meets the Y-axis at B (0, b), therefore we have

$l^2 = a^2 + b^2$

⇒ $(12)^2 = a^2 + b^2$ ...(i) [$\because$ l = 12]



Let P be the point on AB which is 3 cm from A and hence 9 cm from B.
This means that the point P divides AB in ratio 3 : 9 i.e., 1 : 3.
If P = (x, y), then by section formula, we have



(x, y) = $\left(\frac{1\times 0+3\times a}{1+3}, \frac{1\times b+3\times 0}{1+3}\right)$

⇒ (x, y) = $\left(\frac{3a}{4}, \frac{b}{4}\right)$

⇒ x = $\frac{3a}{4}$, y = $\frac{b}{4}$ ⇒ a = $\frac{4x}{3}$ and b = 4y

On putting the values of a and b in Equation (i), we get

144 = $\left(\frac{4x}{3}\right)^2$ + $(4y)^2$

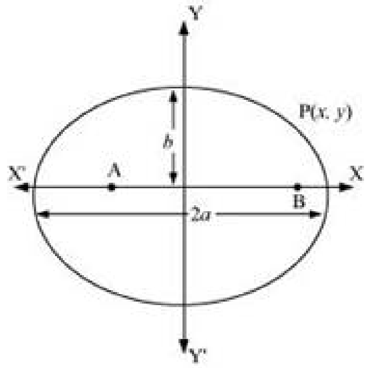
⇒ $\frac{x^2}{81} + \frac{y^2}{9}$ = 1

which is required equation.

173. A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the path traced by the man.

Ans. :

Let A and B be the positions of the two flag posts and P(x, y) be the position of the man.
Accordingly, PA + PB = 10
We know that if a point moves in-plane in such a way that the sum of its distance from two fixed points is constant, then the path is an ellipse and this constant value is equal to the length of the major axis of the ellipse.
Therefore, the path described by the man is an ellipse where the length of the major axis is 10m, while points A and B are the foci.
Taking the origin of the coordinate plane as the center of the ellipse, while taking



the major axis along the x-axis,

The equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.
Accordingly, 2a = 10 $\Rightarrow$ a = 5
Distance between the foci = 2ae = 2c = 8
$\Rightarrow$ c = 4
On using the relation, c = $\sqrt{a^2-b^2}$, we get,
4 = $\sqrt{25-b^2}$
$\Rightarrow$ 16 = 25 – b^2
$\Rightarrow$ b^2 = 25 -1 6 = 9
$\Rightarrow$ b = 3
Put value of a and b in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
$\frac{x^2}{5^2}+\frac{y^2}{3^2}=1 \Rightarrow \frac{x^2}{25}+\frac{y^2}{9}=1$.

174. Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse $9x^2 + 4y^2 = 36$.

Ans. : The given equation of the ellipse can be written in standard form as $\frac{x^2}{4}+\frac{y^2}{9}=1$

Since the denominator of $\frac{y^2}{9}$ is larger than the denominator of $\frac{x^2}{4}$,the major axis is along the y-axis. Comparing the given equation with the standard equation $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we have b = 2 and a = 3.
Also $c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5}$

and $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

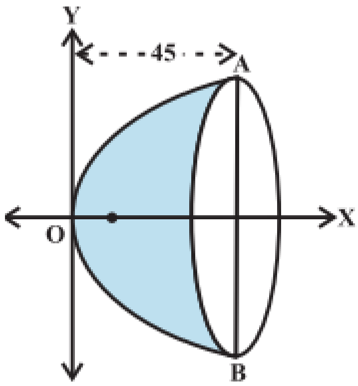
Hence the foci are,$(0, \sqrt{5})$ & $(0, -\sqrt{5})$ vertices are (0,3) & (0, –3),

length of the major axis = 2a = 6 units

the length of the minor axis = 2b = 4 units and

the eccentricity of the ellipse = $\frac{\sqrt{5}}{3}$.

175. The focus of a parabolic mirror as shown in is at a distance of 5 cm from its vertex. If the mirror is 45 cm deep, find the distance AB



Ans. : Since the distance from the focus to the vertex is 5 cm. We have, a = 5. If the origin is taken at the vertex and the axis of the mirror lies along the positive x-axis, the equation of the parabolic section is

y^2 = 4 (5) x = 20 x ==> required eqution of parabola y^2 = 20x

Note that x = 45. Thus

y^2 = 900

Therefore y = $\pm$ 30

Hence AB = 2y = 2 $\times$ 30 = 60 cm

176. If the lines 2x - 3y = 5 and 3x - 4y = 7 are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Ans. : Area of given circle is = 154

$\pi^2 = 154$

$\frac{22}{7}\mathrm{r}^2 154$

$\mathrm{r}^2 = 154 \times \frac{7}{22}$

$\mathrm{r}^2 = 154 \times \frac{7}{22}$

$\mathrm{r}^2 = 49$

$\mathrm{r} = 7$

The intersection point of 2x - 3y = 5 and 3x - 4y = 7 is

The centre of the circle.

Solving simultaneous equations

2x - 3y = 5 and 3x - 4y = 7 we get,

Centre of circle as (1, -1)

Equation of circle with centre (1, -1) and radius= 7 is,

$(x - 1)^2 + (y + 1)^2 = 72$

$x^2 - 2x + 1 + y^2 + 2y + 1 = 49$

$x^2 - 2x + y^2 + 2y = 47$

177. Show that the points (3, -2), (1, 0), (-1, -2) and (1, -4) are concyclic.

Ans. : we have,
P = (3, -2), Q = (1,0), R = (-1,-2) and S = (1, -4)

let us consider A circle $x^2 + y^2 + 2gx + 2fy + c = 0$ .... (1)
Passes through P, Q & R
$\therefore 9 + 4 + 6g - 4f + c = 0$ .... (2)
$1 + 0 + 2g - 0 + c = 0$ .... (3)
$1 + 4 - 2g - 4f + c = 0$ .... (4)
Solving (2), (3) & (4) we get,
g = -1, f = 2 & c = 1
from(1)
The required equation of ercle is

$x^2 + y^2 - 2x + 4y + 1 = 0$ .... (5)
Clearly s = (1, -4) satisfy (5)
Thus,
P, Q, R & S are concydic

178. Find the equation of the circle passing through the points:
(5, -8), (-2, 9) and (2, 1)

Ans. : We know that the general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ .... (1)
We have,
P (5, -8), Q (-2, 9) and R (2, 1)
Since P, Q & R lies on (1)
P, Q & R lies on(1), so,
$25 + 64 + 10g + 16f + c = 0$ .... (2)
$4 + 81 + 4g - 18f + c = 0$ .... (3)
$4 + 1 + 4g - 2f + c = 0$ .... (4)
Solving (2), (3) & (4), we get,
g = 58, f = 24 & c = -285
Thus, equation of eirel e is,

$x^2 + y^2 + 116x - 48y - 285 = 0$ from (1)

179. Find the equation of the circle which passes through the points (3, 7), (5, 5) and has its centre on the line x - 4y = 1.

Ans. : The circle passes through P & Q and the centre lies on
X - 4y = 1 ....(1)

The genral equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ .... (2)

$\therefore$ P & Q lies cn (2), so,

$9 + 49 + 6g + 14f + c = 0$ .... (3)

$25 + 25 + 10g + 10f + c = 0$ .... (4)

Also, centre (-g, -f) lies on (1)

$\therefore$ $- g + 4f = 1$ .... (5)

Solving (3) (4) & (5) we get

g = 3, f = 1 & c = -90

from (2)

The equation of cirde is

$x^2 + y^2 + 6x + 2y - 90 = 0$

180. Find the equation of the circle which circumscribes the triangle formed by the lines

$2x + y - 3 = 0$, $x + y - 1 = 0$ and $3x + 2y - 5 = 0$

Ans. : The given equation of lines

$2x + y = 3$ .... (1)

$x - y = 1$ .... (2)

$3x + 2y = 5$ .... (3)

Let A, B & C are the point of intersection of lines (1) & (2), (2) & (3) and (3) & (1) respectively

$\therefore$ A = (2, -1), B = (3, -2) & C = (1, 1)

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ .... (A)

be the circle that circum scnbinq $\Delta\mathrm{ABC}$

$\therefore$ $4 + 1 - 4g - 2f + c = 0$ .... (4)

$9 + 16 + 6g + 2f + c = 0$ .... (5)

$1 + 1 + 2g + 2f + c = 0$ .... (6)

Solving (4), (5) & (6) we get,

$\mathrm{g} = -\frac{13}{2}, \mathrm{f} = \frac{5}{2} \;\&\; \mathrm{c} = 16$

from (A),

The required cirde is

$x^2 + y^2 - 13x + 5y - 16 = 0$

181. Find the equation of a circle,

Which touches x-axis at a distance 5 from the origin and radius 6 units.

Ans. : The circle touches the x-axis at A = (5, 0) and has radius 6 unit

Thus,

centre = (5, b)

By distance form ulla OA = 6

$\Rightarrow \sqrt{(5-5)^2 + (\mathrm{b}-0)^2} = 6$

$\Rightarrow \mathrm{b}=6$

$\Rightarrow$ Centre $=(5,\ 6)$

so, the equation of required cirde is

$(\mathrm{x}-5)^2+(\mathrm{y}-6)^2=6^2$

$\Rightarrow \mathrm{x}^2+\mathrm{y}^2-10\mathrm{x}-12\mathrm{y}+25=0$

182. Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, latus rectum is 5 and the centre is (0, 0).

Ans. : Let equation of the ellipse be $\frac{\mathrm{x}^2}{\mathrm{a}^2}+\frac{\mathrm{y}^2}{\mathrm{b}^2}=1(\mathrm{a}>\mathrm{b})$

Given that, $\mathrm{e}=\frac{2}{3}$ and latus rectum = 5

$\therefore\ \frac{2\mathrm{b}^2}{\mathrm{a}}=5$

$\Rightarrow \mathrm{b}^2=\frac{5\mathrm{a}}{2}$

We know that, $\mathrm{b}^2=\mathrm{a}^2(1-\mathrm{e})^2$

$\Rightarrow \frac{5\mathrm{a}}{2}=\mathrm{a}^2\left(1-\frac{4}{9}\right)$

$\Rightarrow \frac{5}{2}=\frac{5\mathrm{a}}{9}$

$\Rightarrow \mathrm{a}=\frac{9}{2}$

$\therefore\ \mathrm{b}^2=\frac{5\times9}{2\times2}=\frac{45}{4}$

So, the required equation of the ellipse is $\frac{4\mathrm{x}^2}{81}+\frac{4\mathrm{y}^2}{45}=1$

183. If the line y = mx + 1 is tangent to the parabola y^2 = 4x then find the value of m.
[**Hint:** Solving the equation of line and parabola, we obtain a quadratic equation and then apply the tangency condition giving the value of m]

Ans. : Given that y^2 = 4x ....(i)

and y = mx + 1 ....(ii)

From eq. (i) and (ii) we get

$(mx + 1)^2 = 4x$

$\Rightarrow m^2x^2 + 1 + 2mx - 4x = 0$

$\Rightarrow m^2x^2 + (2m - 4)x + 1 = 0$

Applying condition of tangency, we have

$(2m - 4)^2 - 4m^2 \times 1 = 0$

$\Rightarrow 4m^2 + 16 - 16m - 4m^2 = 0$

$\Rightarrow -16m = -16$

$\Rightarrow m = 1$

Hence, the required value of m is 1

184. Write the coordinate centre of the ellipse $\frac{x^2-ax}{a^2}+\frac{y^2-by}{b^2}=0$

Ans. : $\frac{x^2-ax}{a^2}+\frac{y^2-by}{b^2}=0$

Using completing square method :

$$\Rightarrow \quad \frac{x^2-ax+\frac{a^2}{4}-\frac{a^2}{4}}{a^2}+\frac{y^2-by+\frac{b^2}{4}-\frac{b^2}{4}}{b^2}$$
$$\Rightarrow \quad \frac{\left(x-\frac{a}{2}\right)^2-\frac{a^2}{4}}{a^2}+\frac{\left(y-\frac{b}{2}\right)^2-\frac{b^2}{4}}{b^2}$$
$$\Rightarrow \quad \frac{\left(x-\frac{a}{2}\right)^2}{a^2}-\frac{1}{4}+\frac{\left(y-\frac{b}{2}\right)^2}{b^2}-\frac{1}{4}=0$$
$$\Rightarrow \quad \frac{\left(x-\frac{a}{2}\right)^2}{a^2}+\frac{\left(y-\frac{b}{2}\right)^2}{b^2}=\frac{1}{2}$$
$$\Rightarrow \quad \frac{\left(x-\frac{a}{2}\right)^2}{\left(\frac{a}{\sqrt{2}}\right)^2}+\frac{\left(y-\frac{b}{2}\right)^2}{\left(\frac{b}{\sqrt{2}}\right)^2}=1$$

Let $x-\frac{a}{2}=X, y-\frac{b}{2}=Y$

So,

$$\frac{X^2}{\left(\frac{a}{\sqrt{2}}\right)^2}+\frac{Y^2}{\left(\frac{b}{\sqrt{2}}\right)^2}=1$$

So coordinates of the centre $(0,0)$

$\Rightarrow \quad X=0, Y=0, \quad \text{So}, x=\frac{1}{2}a, y=\frac{1}{2}b$

So coordinates of centre of the ellipse $\left(\frac{a}{2},\frac{b}{2}\right)$

185. Find the coordinates of the centre and radius of the circle $(x\cos\alpha+y\sin\alpha-a)^2+\ (x\sin\alpha-y\cos\alpha-b)^2=k^2$.

Ans. : Equation of given circle is :

$$(x\cos\alpha+y\sin\alpha-a)^2+(x\sin\alpha-y\cos\alpha$$
$$-b)^2=k^2$$
$$(x\cos\alpha+y\sin\alpha)^2-2(x\cos\alpha+y\sin\alpha).$$

$a+a^2+(x\sin\alpha-y\cos\alpha)^2-2\times$
$(x\sin\alpha-y\cos\alpha)\times b+b^2=k^2$
$\Rightarrow x^2\left(\cos^2\alpha+\sin^2\alpha\right)+y^2\left(\sin^2\alpha+\cos^2\alpha\right)+$
$2(b\cos\alpha-a\sin\alpha)\cdot y-2(a\cos\alpha+$
$b\sin\alpha)\cdot x+a^2+b^2-k^2=0$
$\Rightarrow x^2+y^2-2(a\cos\alpha+b\sin\alpha)\cdot x-$
$2(a\sin\alpha-b\cos\alpha)y+a^2+b^2-k^2=0$
Here, $g=-(a\cos\alpha+b\sin\alpha), f=-(a\sin\alpha-$
$b\cos\alpha), c=a^2+b^2-k^2$
i.e. centre of circle $(-g,-f)=[(a\cos\alpha+b\sin\alpha),$
$(a\sin\alpha-b\cos\alpha)]$
and radius of circle $=\sqrt{g^2+f^2-c}$
$=\sqrt{(a\cos\alpha+b\sin\alpha)^2+(a\sin\alpha-b\cos\alpha)^2-a^2-b^2+k^2}$
$$=\sqrt{\begin{array}{c}a^2\cos^2\alpha+b^2\sin^2\alpha+2ab\sin\alpha\cos\alpha+a^2\sin^2\alpha\\+b^2\cos^2\alpha-2ab\sin\alpha\cos\alpha-a^2-b^2+k^2\end{array}}$$
$=\sqrt{a^2\left(\cos^2\alpha+\sin^2\alpha\right)+b^2\left(\sin^2\alpha+\cos^2\alpha\right)-a^2-b^2+k^2}$
$=\sqrt{a^2+b^2-a^2-b^2+k^2}=\sqrt{k^2}=k$

186. If e and e' be the eccentricity of a hyperbola and its conjugate, then prove that $\frac{1}{e^2}+\frac{1}{(e')^2}=1$.

Ans. : Let the equation of the hyperbola be :

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \ldots\ldots(1)$

Equation of its conjugate hyperbola

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \ldots\ldots(2)$

Eccentricity of equation (1)

$e^2=1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$

$e^2=\frac{a^2+b^2}{a^2} \qquad \ldots\ldots(3)$

Eccentricity of equation (2)

$(e')^2=1+\frac{a^2}{b^2}$

$(e')^2=\frac{b^2+a^2}{b^2} \qquad \ldots\ldots(4)$

From equations (3) and (4)

$\frac{1}{e^2}+\frac{1}{e'^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=1$

$\Rightarrow \quad \frac{1}{e^2}+\frac{1}{e'^2}=1$

* **Given section consists of questions of 5 marks each.** **[55]**

187. Find the equation of the circle which passes through the origin and cuts off chords of lengths 4 and 6 on the positive side of the x-axis and y-axis respectively.

Ans. : We have a circle thet passes through origin O (0, 0) and cut of on mtersept of length 4 units on x-axis Be 6 units on y-axis.

That is, OA = 4

OB = 6

C - be the centre of the cirde and CM & CN are perpendicular line drawn on OA & 08 respectively.

Coordinates of A = (4, 0) & B = (0, 6)

$\therefore$ Coordinates of M = (2, 0) & N = (0, 3)

Thus coordinates of C = (2, 3)

Now in $\Delta \mathrm{OCM}$

$OC^2 = OM^2 + CM^2$

$= 2^2 + 3^2$ $[\because$ CM = ON =3 $]$

= 4 + 9

$\therefore \mathrm{OC}=\sqrt{3}$

Thus, the required cirde is

$(x - 2)^2 + (y - 3)^2 = 13$

$x^2 + y^2 - 4X - 6y = 0$

188. Find the equation of the circle, the end points of whose diameter are (2, -3) and (-2, 4). Find its centre and radius.

Ans. : (2, -3) and (-2, 4) are the ends points of the diameter of a circle. The equation of this

circle is (x - 2)(x + 2) + (y + 3)(y - 4) = 0.

$\Rightarrow x^2 - 4 + y^2 - 4y + 3y - 12 = 0$

$\Rightarrow x^2 + y^2 - y - 16 = 0$ .... (1)

Equation (1) can be rewritten as

$\mathrm{x}^2+\left(\mathrm{y}-\frac{1}{2}\right)^2-\frac{1}{4}-16=0$

$\Rightarrow \mathrm{x}^2+\left(\mathrm{y}-\frac{1}{2}\right)^2=\frac{65}{4}$

$\therefore$ Centre is $\left(0,\ \frac{1}{2}\right)$ and radius is $\frac{\sqrt{65}}{2}$.

189. Find the equation of the circle whose diameter is the line segment joining (-4, 3) and (12, -1). Find also the intercept made by it on y-axis.

Ans. : It is given that the end points of the diameter of the circle are (-4, 3) and (12, -1).

∴ Required equation of circle:
(x + 4)(x - 12) + (y - 3)(y + 1)
or $x^2 + y^2 - 8x - 2y - 51 = 0$ ...... (1)
Putting x = 0 in (1):

$y^2 - 2y - 51 = 0$

$\Rightarrow y^2 - 2y - 51 = 0$

$\Rightarrow y = 1 \pm 2\sqrt{3}$

Hence, the intercepts made by it on the y-axis is $1 + 2\sqrt{3} - 1 + 2\sqrt{13} = 4\sqrt{13}$

190. The sides of a square are x = 6, x = 9, y = 3 and y = 6. Find the equation of a circle drawn on the diagonal of the square as its diameter.

Ans. : Let the sides AB, BC, CD and DA of the square ABCD be represented by the equations
y = 3, x = 6, y = 6 and x = 9 respectively.
Then, coordinates are
A(6, 3), B(9, 3), C(9, 6) and D(6, 6).
The equation of the circle with diagonal AC
(x - 6)(x - 9) + (4 - 3)(4 - 6) = 0

$\Rightarrow x^2 - 6x - 9x + 54 + y^2 - 3y - 6y + 18 = 0$

$\Rightarrow x^2 + y^2 - 15x - 9y + 72 = 0$

The equation of the circle with diagonal BD as diameter is
(x - 9)(x - 6) + (y - 3) (y - 6) =

$\Rightarrow x^2 - 9x - 6x + 54 + y^2 - 3y - 6y + 18 = O$

$\Rightarrow x^2 + y^2 - 15x - 9y + 72 = O$

$x^2 + y^2 - 15x - 9y + 72 = O$

191. Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = a\left(\frac{1-t^2}{1+t^2}\right)$2 lies on a circle for all real values of t such that $-1 \le t \le 1,$ where a is any given real number.

Ans. : $x = \frac{2at}{1+t^2}, y = a\left(\frac{1-t^2}{1+t}\right)$

$x^2 + y^2 = \frac{4a^2t^2}{(1+t^2)^2} + \frac{a^2(1-t^2)^2}{(1+t^2)^2}$

$= \frac{4a^2t^2+a^2(1-2t^2+a^2)t^4}{(1+t^2)^2}$

$= \frac{4a^2t^2+a^2-2a^2t^2+a^2t^4}{(1+t^2)^2}$

$= \frac{2a^2t^2+a^2+a^2t^2}{(1+t^2)^2}$

$= \frac{a^2(1+2t^2+t^2)}{(1+t^2)^2}$

$x^2 + y^2 = a^2$ is equation of a circle.

192. Prove that the radii of the circles $x^2 + y^2 = 1$, $x^2 + y^2 - 2x - 6y - 6 = 0$ and $x^2 + y^2 - 4x - 12y - 9 = 0$ are in A.P.

Ans. : The given equation of circle are.

$x^2 + y^2 = 1$ .... (1)

$x^2 + y^2 + 2x + 6y - 6 = 0$ .... (2)

$x^2 + y^2 - 4x - 12y - 9 = 0$ .... (3)

Respectively If a, b, care in AP, then b $\frac{a+c}{2}$

Let C_1 C_2 & C_3 are the centres of (1) (2) & (3)

For a = 1, b = 4, c = 7, $\frac{1+7}{2} = 4 \text{ b},$ therefore 1, 4, 7 or The centres of the three circles lie In AP.

$\therefore$ $R_1 = 1$

$R_2 = \sqrt{g^2 + f^2 - c} = \sqrt{1^2 + 3^2 + 6} = \sqrt{16} = 4$

$R_3 = \sqrt{g^2 + f^2 + c} = \sqrt{2^2 + 6^2 + 9} = \sqrt{49} = 7$

193. Find the vertex, focus, axis, directrix and latus-rectum of the following parabolas: $4(y - 1)^2 = -7(x - 3)$.

Ans. : The given equation is

$4(y-1)^2 = -7(x-3)$

$\Rightarrow (y-1)^2 = \frac{-7}{4}(x-3)\ldots(i)$

Shifting the origin to the point (3, 1) without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y, we have,

$x = X + 3,\ y = Y + 1 \text{....(ii)}$

Using these relation (i), redus to

$Y^2 = \frac{-7}{4}X\ldots\ldots(iii)$

This is of the form $Y = -4aX$, on comparing, we get

$4a = \frac{7}{4}$

$\Rightarrow a = \frac{7}{16}$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0,\ Y = 0)$

$\therefore\ x = 0 + 3,\ y = 0 + 1$ [Using equation (iii)]

$\Rightarrow x = 3,\ y = 1$

$\therefore$ coordinate of the vertex w.r.t new axes are (3, 1).

Focus: The coordinate of the focus w.r.t new axes are $\left(x = -\frac{7}{16}, y = 0\right)$

$\therefore x - \frac{-7}{16} + 3,\ y = 0 + 1$

$\Rightarrow x = \frac{41}{16},\ y = 1$

$\therefore$ coordinates of the focus w.r.t old axes are $\left(\frac{41}{16}, 1\right)$

Axis: Equation of the axes of the parabola w.r.t new axes is

$y = 0$

$\Rightarrow y = 0 + 1$

$\Rightarrow y = 1$

$\therefore$ equation of axis w.r.t old axes is $y = 1$

Directrix: Equation of the directrix of the parabola w.r.t new axes is

$Y = \frac{7}{16}$

$\therefore x = \frac{7}{16} + 3$

$\Rightarrow x = \frac{55}{16}$

$\therefore$ Equation of the directrix of the parabola w.r.t oid axes is $x = \frac{55}{16}$.

Latus-rectum: The length of the latus-rectum = 4a

$= 4 \times \frac{7}{16}$

$= \frac{7}{4}.$

194. Find the equation of the parabola, if
The focus is at (0, -3) and the vertex is at (-1, -3).

Ans. : In a parabola, vertex is the mid-point of the focus and the point of the intersection of the axis and directrix. So, let (x_1, y_1) be the co-ordinate of the point of intersection of the axis and directrix.

Then (-1, -3) is the mid-point of the line segment joining (0, -3) and (x_1, y_1).

$\therefore \frac{x_1+0}{2} = -1$ and $\frac{y_1-3}{2} = -3$

$\Rightarrow x_1 = -2$ and $y_1 = -3$

Thus, the directrix meets the axis at (-2, -3)

Let A be the vertex and S be the focus of the required parabola.

Then,

$m_1 = \text{slope of AS} = \frac{-3-(-3)}{0-(-1)} = 0$

$\therefore$ slope of the directrix $= \frac{-1}{0} = \infty$

Thus, the directrix passes through (-2, -3) and has slope ∞, so its equation is

$y - (-3) = \infty(x - (-2))$

$\frac{y+3}{\infty} = x + 2$

$\Rightarrow x + 2$

Let P(x, y) be a point on parabola.

Then, PS = Distance of P from the directrix.

$\sqrt{(x-2)^2 + (y+3)^2} = \left|\frac{x+2}{\sqrt{1^2}}\right|$

$\Rightarrow x^2 + (y+3)^2 = (x+2)^2$

$\Rightarrow x^2 + y^2 + 9 + 6y = x^2 + 4 + 4x$

$\Rightarrow y^2 - 4x + 6y + 9 - 4 = 0$

$\Rightarrow y^2 - 4x + 6y + 5 = 0.$

195. Find the equation of the set of all points the sum of whose distances from the points (3, 0) and (9, 0) is 12.

Ans. : Let (x, y) be any point.
Given points are (3, 0) and (9, 0) is 12
According to the quation, we have

$$\sqrt{(x-3)^2+(y-0)^2}+\sqrt{(x-9)^2+(y-0)^2}=12$$

$$=\sqrt{x^2+9-6x+y^2}+\sqrt{x^2+81+18x+y^2}=12$$

Putting $x^2+9-6x+y^2=k$

$$\Rightarrow \sqrt{k}+\sqrt{72-12x+k}=12$$

$$\Rightarrow \sqrt{72-12x+k}=12-\sqrt{k}$$

Squaring sides, we have

$$\Rightarrow 72-12x+k=144+k-24\sqrt{k}$$

$$\Rightarrow 24\sqrt{k}=144-72+12x$$

$$\Rightarrow 24\sqrt{k}=72+12x$$

$$\Rightarrow 2\sqrt{k}=6+x$$

Again squaring both sides, we get

$$4k=36+x^2+12x$$

Putting the value of k, we get

$$4(x^2+9-6x+y^2)=36+x^2+12x$$

$$\Rightarrow 4x^2+36-24x+4y^2=36+x^2+12x$$

$$\Rightarrow 3x^2+4y^2-36x=0$$

Hence, the required equation is $3x^2 + 4y^2 - 36x = 0$

196. If the lines 2x - 3y = 5 and 3x - 4y = 7 are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Ans. : We know that the intersection point of the diameter gives the centre of the circle.
Given equation of diameters are
2x - 3y = 5 ....(i)
3x - 4y = 7 ......(ii)

From eq. (i) we have $x=\frac{5+3y}{2}\ldots(\text{iii})$
Putting the value of x in eq. (ii) we have

$$3\left(\frac{5+3y}{2}\right)-4y=7$$

$$\Rightarrow 15+9y-8y=14$$

$$\Rightarrow y=14-15$$

$$\Rightarrow y=-1$$

Now, from eq. (iii) we have

$$x=\frac{5+3(-1)}{2}$$

$\Rightarrow x = \frac{5-3}{2}$

$\Rightarrow x = 1$

So, the centre of the circle = (1, -1)

Given that area of the circle = 154

$\Rightarrow \pi r^2 = 154$

$\Rightarrow \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = 154 \times \frac{7}{22}$

$\Rightarrow r^2 = 7 \times 7$

$\Rightarrow r = 7$

So, the equation of the circle is,

$\Rightarrow (x-1)^2 + (y+1)^2 = (7)^2$

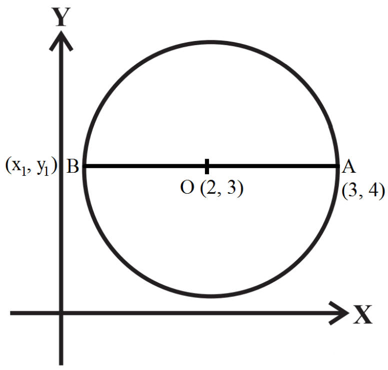
$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

Hence, required of the circle is,

$x^2 + y^2 - 2x + 2y = 47$

197. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinate of the other end of the diameter.



Ans. :

Equation of given circle is $x^2 + y^2 - 4x - 6y + 11 = 0$

Centre = (-g, -f) = (2, 3)

$\therefore \frac{x_1+3}{2} = 2$

$\Rightarrow x_1 + 3 = 4$

$\Rightarrow x_1 = 1$

and $\frac{y_1+4}{2} = 3$

$\Rightarrow y_1 + 4 = 6$

$\Rightarrow y_1 = 2$

Hence, the required coordinates are (1, 2).

----- जब तक किसी काम को हम शुरू नहीं करते,तब तक वह नामुमकिन ही लगता है| -----